Some Issues on Self-Calibration and Critical Motion Sequences

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ABSTRACT

This paper is concerned with the problem of self-calibration. First some general concepts are discussed, then several methods are briefly discussed. In the second part of the paper the problem of critical motion sequences is treated. This problem could cause an important limitation to the practical use of self-calibration. In this context several interesting results are presented.

Keywords: self-calibration, critical motion sequences, absolute conic, ambiguous reconstruction.

1. INTRODUCTION

Since it became clear that it was possible to obtain a projective reconstruction (i.e. a reconstruction determined up to an arbitrary projective transformation) from a set of uncalibrated images of a scene [4, 7], researchers have tried to obtain ways to upgrade this reconstruction to a metric one (i.e. determined up to an arbitrary euclidean transformation and a scalefactor). In general three types of constraints could be applied to achieve this: scene constraints, camera motion constraints and constraints on the camera intrinsics. All of these have been tried separately or in conjunction. Reducing the ambiguity on the reconstruction by imposing restricitions on the camera intrinsic camera parameters is termed self-calibration (in the area of computer vision). In recent years many researchers have been working on this subject. Mostly self-calibration algorithms are concerned with unknown but constant intrinsic camera parameters (see for example Faugeras et al. [5], Hartley [8], Pollefeys and Van Gool [20, 24], Heyden and Åström [11] and Triggs [33]). Recently, the problem of self-calibration in the case of varying intrinsic camera parameters was also studied (see Pollefeys et al. [21, 28, 26] and Heyden and Åström [12, 13]).

Many researchers proposed specific self-calibration algorithms for restricted motions (i.e. combining camera motions constraints and constraints on the camera intrinsics). In several cases it turns out that simpler algorithms can be obtained. The price to pay is, however, that the ambiguity can often not be restricted to metric. Some interesting approaches were proposed by Moons et al. [19] for pure translation, Hartley [9] for pure rotations and by Armstrong et al. [1] (see also [6]) for planar motion.

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Recently, some methods were proposed to combine self-calibration with scene-constraints. A specific combination was proposed in [23] to resolve a case with minimal information. Bondyfalat and Bougnoux [2] proposed a method of elimination to impose the scene constraints. Liebowitz and Zisserman [16], on the other hand, formulate both the scene constraints and the self-calibration constraints as constraints on the absolute conic, so that a combined approach is achieved.

Another important aspect of the self-calibration problem is the problem of critical motion sequences. In some cases the motion of the camera is not general enough to allow for self-calibration and an ambiguity remains on the reconstruction. A first complete analysis for constant camera parameters was given by Sturm [30]. Others have also worked on the subject (e.g. Pollefeys [26], Ma et al. [18] and Kahl [14]).

This paper is organized as follows. Section 2 discusses the self-calibration problem. The general concepts are presented and some specific methods are discussed. Section 3 discusses the problem of critical motion sequences. In this context some bad news and some good news is given. The paper is summarized in Section 4.

2. SELF-CALIBRATION

In this section some importants concepts for self-calibration are introduced. These are then used to briefly described most of the existing self-calibration methods for general motions.

The image of the absolute conic

One of the most important concepts for self-calibration is the Absolute Conic (AC) and its projection in the images (IAC). Since it is invariant under Euclidean transformations, its relative position to a moving camera is constant. For constant intrinsic camera parameters its image will therefore also be constant. This is similar to someone who has the impression that the moon is following him when driving on a straight road. Note that the AC is more general, because it is not only invariant to translations but also to arbitrary rotations.

It can be seen as a calibration object which is naturally present in all the scenes. Once the AC is localized, it can be used to upgrade the reconstruction to metric. It is, however, not always so simple to find the AC in the reconstructed space. In some cases it is not possible to make the difference between the true AC and other candidates. This problem will be discussed in the Section 3.

In practice the simplest way to represent the AC is through the Dual Absolute Quadric (DAQ). In this case both

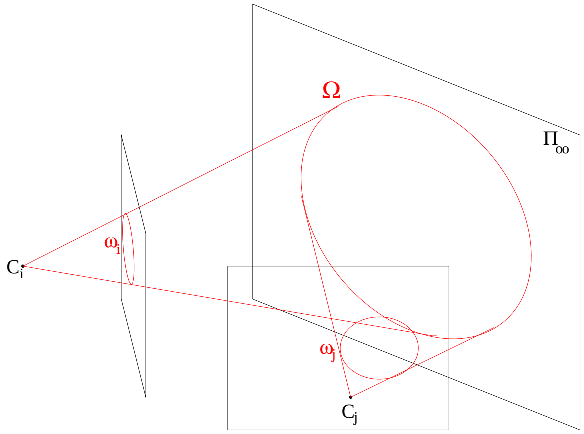

Figure 1. *The absolute conic (located in the plane at infinity) and its projection in the images*

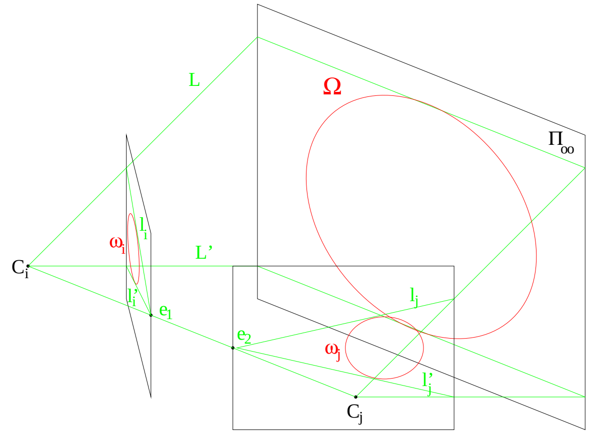

Figure 2. *The Kruppa equations impose that the image of the absolute conic satisfies the epipolar constraint. In both images the epipolar lines corresponding to the two planes through $\mathtt{C}_i$ and $\mathtt{C}_j$ tangent to Ω must be tangent to the images ω_i and ω_j.*

the AC and its supporting plane, the plane at infinity, are expressed through one geometric entity. The relationship between the AC and the IAC is easily obtained using the projection equation for the DAQ:

$$\omega_i^* \sim \mathbf{P}_i \Omega^* \mathbf{P}_i^\top \quad . \tag{1}$$

with ω_i^* representing the dual of the IAC, Ω^* the DAQ and $\mathbf{P}_i$ the projection matrix for view i. Figure 1 illustrates these concepts. For a Euclidean representation of the world the camera projection matrices can be factorized as: $\mathbf{P}_i = \mathbf{K}_i \mathbf{R}_i^\top [\mathbf{I}|\text{-}\mathtt{t}_i]$ (with $\mathbf{K}_i$ an upper triangular matrix containing the intrinsic camera parameters, $\mathbf{R}_i^\top$ representing the orientation and $\mathtt{t}_i$ the position) and the DAQ can be written as $\Omega^* = diag(1,1,1,0)$. Substituting this in eq.(1), one obtains:

$$\omega_i^* \sim \mathbf{K}_i \mathbf{K}_i^\top \tag{2}$$

This equation is very useful, because it immediately relates the intrinsic camera parameters to the DIAC.

In the case of a projective representation of the world the DAQ will not be at its standard position, but will have the following form: $\Omega^* = \mathbf{T}\Omega_M^* \mathbf{T}^\top$ with $\mathbf{T}$ being the transformation from the metric to the projective representation. But, since the images were obtained in a Euclidean world, the images ω_i^* still satisfies (2). If Ω^* is retrieved, it is possible to upgrade the geometry from projective to metric.

The IAC can also be transferred from one image to another through the homography of its supporting plane (i.e. the plane at infinity):

$$\omega_j \sim \mathbf{H}_{ij}^{\infty\,-\top} \omega_i \mathbf{H}_{ij}^{\infty\,-1} \text{ or } \omega_j^* \sim \mathbf{H}_{ij}^{\infty} \omega_i^* \mathbf{H}_{ij}^{\infty\,\top} \quad . \tag{3}$$

It is also possible to restrict this constraint to the epipolar geometry. In this case one obtains the Kruppa equations [15] (see Figure 2):

$$[\mathtt{e}_{ij}]_\times^\top \mathbf{K}\mathbf{K}^\top [\mathtt{e}_{ij}]_\times \sim \mathbf{F}_{ij} \mathbf{K}\mathbf{K}^\top \mathbf{F}_{ij}^\top \tag{4}$$

with $\mathbf{F}_{ij}$ the fundamental matrix for views i and j and $\mathtt{e}_{ij}$ the corresponding epipole. In this case only 2 (in stead of 5) independent equations can be obtained [34]. In fact restricting the self-calibration constraints to the epipolar geometry is equivalent to the elimination of the position of infinity from the equations. The result is that some artificial degeneracies are created (see [32]).

Self-calibration methods

Here some self-calibration approaches are briefly discussed. Combining eq.(1) and (2) one obtains the following equation:

$$\mathbf{K}_i \mathbf{K}_i^\top \sim \mathbf{P}_i \Omega^* \mathbf{P}_i^\top \tag{5}$$

Several methods are based on this equation. For constant intrinsic parameters Triggs [33] proposed to mimimize the deviation from eq.(5). A similar approach was proposed by Heyden and Åström [11]. Pollefeys and Van Gool [24] proposed a related approach based on the transfer equation (i.e. eq. (3)) rather than the projection equation. These different approaches are very similar as was shown in [24]. The more flexible self-calibration method which allows varying intrinsic camera parameters [28] is also based on this equation.

The first self-calibration method was proposed by Faugeras et al. [5] based on the Kruppa equations (eq.(4)). The approach was impoved over the years [17, 34]. An interesting feature of this self-calibration technique is that no consistent projective reconstruction should be available, only pairwise epipolar calibration. This can be very useful is some cases where it is hard to relate all the images into a single projective frame. The price that is paid for this advantage is that 3 of the 5 absolute conic transfer equations are used to eliminate the dependence on the position of the plane at infinity. This explains why this method performs poorly compared to others when a consistent projective reconstruction can be obtained (see [27]).

When the homography of the plane at infinity $\mathbf{H}_{ij}^\infty$ is known, then eq.(3) can be reduced to a set of linear equations in the coefficients of ω_i or ω_i^* (this was proposed by Hartley [8]). Several self-calibration approaches rely on this possibility. Some methods follow a stratified approach and obtain the homographies of the plane at infinity by first reaching an affine calibration, based an a pure translation (see Moons et al. [19]) or using the modulus constraint (see Pollefeys et al. [27]). Other methods are based on pure rotations (see Hartley [9] for constant intrinsic parameters and de Agapito et al. [3] for a zooming camera).

A few years ago Hartley proposed an alternative self-calibration method [8]. This method is not based on the absolute conic, but directly uses a QR-decomposition of the camera projection matrices. Hartley uses the following equation:

$$\mathbf{P}_i \begin{bmatrix} \mathbf{I} \\ \pi_\infty^\top \end{bmatrix} \mathbf{K} \sim \mathbf{K}_i \mathbf{R}_i \qquad (6)$$

where $\mathbf{K}$ and π_∞ are the unknowns. It is proposed to compute $\mathbf{K}_i$ through QR-decomposition of the left-hand side of eq.(6). The following equation should be roughly satisfied for the solution:

$$\mathbf{K} \approx \mathbf{K}_i \text{ or } \mathbf{K}^{-1}\mathbf{K}_i \approx \mathbf{I} \qquad (7)$$

The main difference between Hartley's method and the other is that the rotational component is eliminated through QR-decomposition instead of through multiplication by the transpose.

In fact once the metric reconstruction has been obtained through self-calibration it is indicated to refine the results through a maximum likelihood approach, i.e. *bundle adjustment*. This is a standard technique in photogrammetry [29] and is also more and more used in computer vision nowadays. Traditionally several assumptions are made in this case. It is assumed that the error is only due to mislocalization of the image features. Additionally, this error should be uniformly and normally distributed[1]. This means that the proposed camera model is supposed to be perfectly satisfied. In these circumstances the maximum likelihood estimation corresponds to the solution of a least-squares problem:

$$\mathcal{C}_{ML} = \sum_{i=1}^{n} \sum_{l \in I_i} \left((x_{li} - \frac{\mathtt{P}_{i1}\mathtt{M}_l}{\mathtt{P}_{i3}\mathtt{M}_l})^2 + (y_{li} - \frac{\mathtt{P}_{i2}\mathtt{M}_l}{\mathtt{P}_{i3}\mathtt{M}_l})^2 \right)$$

where I_i is the set of indices corresponding to the points seen in view i and $\mathbf{P}_i \equiv \left[\mathtt{P}_{i1}^\top \mathtt{P}_{i2}^\top \mathtt{P}_{i3}^\top\right]^\top = \mathbf{K}_i[\mathbf{R}_i^\top | \text{-}\mathbf{R}_i^\top \mathbf{t}_i]$. In this equation $\mathbf{K}_i$ should be parameterized so that the self-calibration constraints are satisfied. The model could also be extended to deal with more complex camera models (e.g. radial distortion).

In [12] it was even proposed to use bundle adjustment immediately. Since the subject of initialization was not covered in this paper, questions can be raised concerning the practical feasability of this approach. If one can obtain a good initialization, however, bundle adjustment can significantly improve the final results.

3. CRITICAL MOTION SEQUENCES

It was noticed very soon that not all motion sequences are suited for self-calibration. Some obvious cases are the restricted motions described in the previous section (i.e. pure translation, pure rotation and planar motion). There are, however, more motion sequences which do not lead to unique solutions for the self-calibration problem. This means that at least two reconstructions are possible which satisfy all constraints on the camera parameters for all the images of the sequence and which are not related by a similarity transformation.

[1] This is a realistic assumption since outliers should have been removed at this stage of the processing.

Figure 3. *Some images of the Arenberg castle*

Several researchers realized this problem and mentioned some specific cases or did a partial analysis of the problem [33, 34, 22]. Sturm [30, 31] provided a complete catalogue of critical motion sequences (CMS) for constant intrinsic parameters. Additionally, he identified specific degeneracies for some algorithms [32].

It is, however, very important to notice that the classes of CMS that exist depend on the constraints that are enforced during self-calibration. The extremes being all parameters known, in which case almost no degeneracies exist; and, no constraints at all, in which case all motion sequences are critical.

It is outside the scope of this paper to give an analysis which leads to the exhaustive list of CMS for the different possible sets of constraints. For some specific cases we refer to the work of Sturm [30, 32], Pollefeys [26], Kahl [14] and Ma [18].

All implications of CMS are not yet fully understood. Some results seem to show that the problem is worse than what could be expected at first, but on the other hand some results are also encouraging.

Some bad news

It is clear that in practice motion sequences will almost never be perfectly critical. One can however expect that some sequences will be quasi-critical and thus lead to ambiguous reconstruction in the presence of noise. The question is thus: *How far should a motion sequence be from critical to allow a good self-calibration?* Of course, this question is not so simple. What does "*far*" from a class of critical motions mean in the space of all possible motions? The following experiment, however, gives some intuitive insight that this could be further than expected.

A sequence of images of the Arenberg castle was recorded. In Figure 3 some of the images of the sequence are shown. A 3D reconstruction was obtained based on the method described in Pollefeys et al. [25]. The reconstruction is illustrated together with the retrieved camera position and orientation in Figure 4 The self-calibration method assumes that all intrinsic camera paramaters are known, except the focal length which is free to vary [28].

The linear algorithm provides a good result, even though the motion sequence is close to critical with respect to this algorithm[2]. This solution is used as the initialization for the nonlinear algorithm. This algorithm converges without problems for the case of a varying focal length. After 6 iterations the norm of the residue vector is reduced from 0.57 to 0.11. For both algorithms the retrieved focal length is almost constant, as it should be. This constant value however differs for

[2] The reconstructed motion sequence (see Figure 4) almost keeps the point in the center of the image fixed.

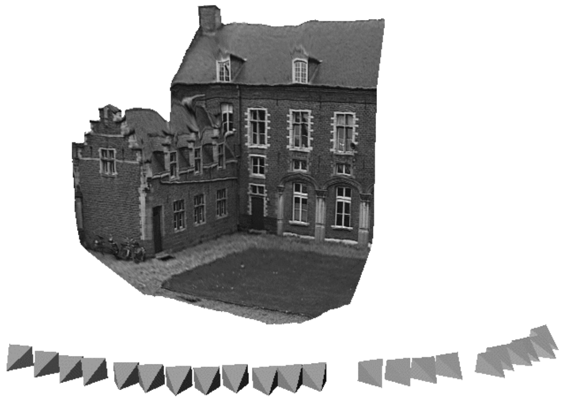

Figure 4. *Perspective view of the reconstruction together with the estimated position of the camera for the different views of the sequence*

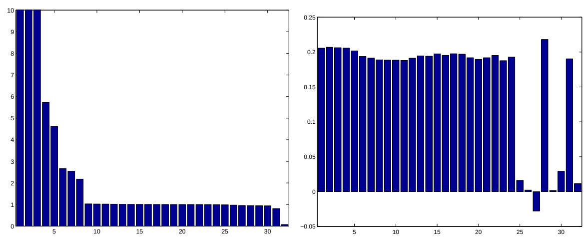

Figure 5. *Structure of the jacobian. Singular values (left) and right singular vector associated with the smallest singular value (right). The first 3 singular values were clipped, the values are 42, 39 and 31. The first 24 unknowns are associated with the focal lengths, the next 3 with the position of the plane at infinity and the last 5 with the absolute conic.*

both algorithms. The reason for this is explained based on the analysis of the jacobian of the self-calibration equations at the solution.

On the left side of Figure 5 the singular values of this jacobian are given. Note that the last one is much smaller than the others. The associated singular vector is shown on the right of this figure. It indicates that it is not possible to accurately determine the absolute value of the focal length from this specific image sequence. Note, however, that an orbital motion –to which the actual motion of the camera is very close– is not critical in this case. The problem is that the angle of rotation between the extreme views of the castle sequence is too small to allow for an accurate self-calibration. This was verified with synthetic data. Two sequences of orbital motion were generated. One of 60 degrees (similar to the castle sequence) and one of 360 degrees. The first sequences also yielded a jacobian with a small singular value. The jacobian of the second sequence did not have any small singular value. In fact, in terms of self-calibration, the castle sequence is relatively close to pure translation (although the orientation change is almost 60 degrees!). This is bad news!

Some good news

The classification of all possible critical motion sequences for a specific set of self-calibration constraints can be used to avoid critical motions when acquiring an image sequence on which one intends to use self-calibration. In some cases, however, an uncalibrated image sequence is available from which a metric reconstruction of the recorded scene is expected. In this case, it is not always clear, at first, what can be achieved nor if the motion sequence is critical or not.

It can be shown that the recovered motion sequence for any reconstruction satisfying the fixed absolute conic image constraint would consist of rigid motions (i.e. Euclidean motions). This result is also valid for critical motion sequences, where the recovered motion sequence would be in the same CMS class as the original sequence. In [31] a proof was given for the case of constant intrinsic camera parameters. Here we give a simpler and more general proof which is based on the disc quadric representation. It is valid for all possible types of self-calibration constraints.

Theorem 1 *Let S be a motion sequence that is critical with respect to the dual quadric Φ^*, and let $\mathbf{P}_{Ei}$ be the original projection matrices of the frames in S. Let $\mathbf{T}$ be any projective transformation mapping Φ^* to Ω^* and $\mathbf{P}_{Pi} = \mathbf{P}_{Ei}\mathbf{T}^{-1}$ be the projection matrices transformed by $\mathbf{T}$. There exists a Euclidean transformation between any pair of $\mathbf{P}_{Pi}$.*

Proof: From S being a critical motion sequence with respect to Φ^*, it follows that there must exist a $\mathbf{K}_{P}i$ which respects the self-calibration constraints for which

$$\mathbf{K}_{Pi}\mathbf{K}_{Pi}^\top \sim \phi_i^* \sim \mathbf{P}_{Ei}\Phi^*\mathbf{P}_{Ei}^\top$$

Since $\Phi^* \sim \mathbf{T}^{-1}\Omega^*\mathbf{T}^{-\top}$ and $\mathbf{P}_{Pi} = \mathbf{P}_{Ei}\mathbf{T}^{-1}$, one gets

$$\mathbf{K}_{Pi}\mathbf{K}_{Pi}^\top \sim \mathbf{P}_{Pi}\Omega^*\mathbf{P}_{Pi}^\top$$

Defining $\mathbf{H}_{Pi}$ as the left 3×3 part of $\mathbf{P}_{Pi}$ this yields

$$\mathbf{K}_{Pi}\mathbf{K}_{Pi}^\top \sim \mathbf{H}_{Pi}\mathbf{H}_{Pi}^\top \text{ or } \mathbf{I} \sim \mathbf{K}_{Pi}^{-1}\mathbf{H}_{Pi}\mathbf{H}_{Pi}^\top\mathbf{K}_{Pi}^{-\top}$$

Since the matrix $\mathbf{K}_{Pi}^{-1}\mathbf{H}_{Pi}$ satisfies the orthonormality constraints, it must correspond to some rotation matrix, say $\mathbf{R}_{Pi}$. Thus $\mathbf{H}_{Pi} = \mathbf{K}_{Pi}\mathbf{R}_{Pi}$. Therefore, it is always possible to write the projection matrices $\mathbf{P}_{Pi}$ as follows:

$$\mathbf{P}_{Pi} = \mathbf{K}_{Pi}[\mathbf{R}_{Pi}^\top \,|\, \text{-}\mathbf{R}_{Pi}^\top\mathbf{t}_{Pi}]$$

□

This means that the sequence S_P consisting of $\mathbf{P}_{Pi}$ is Euclidean and has, after transformation by $\mathbf{T}$, the same set of potential absolute conics. Since the different classes of CMS given in [30] can't be transformed into each other through a projective transformation, the sequence S_P will be a CMS of the same class as S. Therefore, one can conclude that any reconstruction being a solution to the self-calibration problem allows us to identify the class of CMS of the original sequence and thus also all ambiguous reconstructions. This is an important observation, because it allows to identify critical motion sequences and to determine the ambiguity on the reconstruction from any valid instantiation of the reconstruction.

In that case more specific algorithms can be called or additional constraints can be brought in to reduce the ambiguity [35]. In addition, the values for the intrinsic camera parameters are not arbitrary. The pixels are always rectangular and most often close to squares. The principal point is in general close to the center of the image and even the focal length can not take on arbitrary values. On top of that one can impose that all the visible points must be located in front of the camera (see [10]). This means that even for critical motion sequences one will in general be able to do much better than what could be expected from the CMS analysis.

In some case the reconstruction will thus be an ambiguous one. An important question is then: *What can still be done with an ambiguous reconstruction?* An interesting answer is given by the next theorem (a similar result was very recently presented by Ma [18]).

Let us first define $C(S)$ as being the set of potential absolute quadrics for the motion sequences S. Let us also define the transformation of S as the sets of the transformed elements.

$$S = \{ \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0_3^\top & 1 \end{bmatrix}, \ldots \} \rightarrow \mathbf{T}S = \{ \mathbf{T} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0_3^\top & 1 \end{bmatrix} \}$$

Theorem 2 *Let S be a critical motion sequence and $\mathbf{P}_{Ei}$ the corresponding projection matrices. Let Φ^* be an arbitrary element of $C(S)$ and let $\mathbf{T}$ be an arbitrary projective transformation mapping Φ^* to Ω^*. Let $S_P = \mathbf{T}S$ and $\mathbf{P}_{Pi} = \mathbf{P}_{Ei}\mathbf{T}^{-1}$. Let M represent a Euclidean motion for which $C(S_P \cup M) = C(S_P)$ and let $\mathbf{P}_{Pnew}$ be the corresponding projection matrix. Then there exists a Euclidean transformation between $\mathbf{P}_{Enew} = \mathbf{P}_{Pnew}\mathbf{T}$ and any other $\mathbf{P}_{Ei}$.*

Proof: From $\Omega^* \in C(S)$, it follows that $\mathbf{T}\Omega^*\mathbf{T}^\top \in C(S_P)$. Since it is assumed that $C(S_P \cup M) = C(S_P)$, it follows that Theorem 1 can be applied to the sequence $S_P \cup M$, with the dual quadric $\mathbf{T}\Omega^*\mathbf{T}^\top$, the transformation $\mathbf{T}^{-1}$ and $\{\mathbf{P}_{P1}, \ldots, \mathbf{P}_{Pn}, \mathbf{P}_{Pnew}\}$ as so-called original projection matrices. □

This theorem allows us to conclude that it is possible to generate correct new views, even starting from an ambiguous reconstruction. In this case, we should, however, restrict the motion of the virtual camera to the type of the critical motion sequence recovered in the reconstruction. For example, if we have acquired a model by doing a planar motion on the ground plane and thus rotating around vertical axes, then we should not move the camera outside this plane nor rotate around non-vertical axes. But, if we restrict our virtual camera to this critical motion, then all these motions will correspond to Euclidean motions in the real world and no distortion will be present in the images (except, of course, for modeling errors). Note that the recovered camera parameters should be used[3], except when some parameters can vary (in which case any possible parameter setting can be used). In fact, this result is related to the more general rule that for the generation of new views interpolation is more desirable than extrapolation.

[3] the ones obtained by factorizing $\mathbf{P}_{Pi}$ in $\mathbf{K}_{Pi}[\mathbf{R}_{Pi}^\top | \text{-}\mathbf{R}_{Pi}^\top \mathbf{t}_{Pi}]$.

4. CONCLUSION

In this paper the absolute conic was presented as a central concept for self-calibration. This concept is very usefull for practical algorithms as well as for a theoretical analysis of the problem. An important number of existing self-calibration methods were briefly discussed. Next, the important problem of the critical motion sequences (CMS) was treated. Although in practice exact CMS are very improbable to occure, many motion sequences seem to be quasi-critical. This puts an important limitation on the use of self-calibration for restricted motion sequences. On the other hand it seems that it is possible to deal with CMS. It was shown that it is possible to detect their occurence and therefore to deal with the ambiguity. For the generation of new views it seems that –taking some restrictions into account– even from an ambiguous reconstruction undistorted new views can be generated.

ACKNOWLEDGEMENTS

The financial support of the IWT ITEA-project BEYOND is gratefully acknowledged.

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