ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Polyhedral Computation, Spring 2016
Assignment 3*

March 23, 2016

Problem 1 (Hilbert Basis): Show that if a rational cone is not pointed, a minimal integral Hilbert basis is not unique.

Problem 2 (Lower bound on the Size of a Hilbert Basis): In the proof of Theorem 4.1, assume that $t = n$ and the rational cone is generated by n linearly independent vectors $C = \text{cone}(\{a_1, a_2, \ldots, a_n\})$. Derive a tight lower bound of k in terms of n and the absolute value of the determinant $\det([a_1, a_2, \ldots, a_n])$. Note that k is the number of lattice points in the zonotope Z and $k \geq 2^n$, because the zonotope Z is combinatorially a cube.

Problem 1 (Convexity Checking): Consider any collection $P_1, P_2, \ldots, P_k$ of convex polytopes. Each polytope P_i is given as a system of linear inequalities $A^i x \leq b^i$ and as the set V^i of extreme points. The decision problem of checking the convexity of the union $P_1 \cup P_2 \cup \ldots \cup P_k$ is coNP-complete. Give a mixed IP formulation which attains a positive optimal value if and only if the union is not convex.

*Please hand in your solution to May Szedlák CAB G19.2 (may.szedlak@inf.ethz.ch) no later than **Tuesday, April 12, 2016**.