ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Polyhedral Computation, Spring 2016
Assignment 2*

March 09, 2015

Problem 1 (Fourier-Motzkin Elimination): Consider a system of linear inequalities $Ax \leq b$ where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$. Apply Fourier-Motzkin Elimination in the i'th variable. Denote the resulting matrix with A', the right-hand side with b'.

1. Prove that $Ax \leq b \Leftrightarrow A'x' \leq b'$ where $x' = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)^T$.
2. Determine whether the following two systems of inequalities admit a feasible solution.

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & -1 \\ -5 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 4 \\ 5 \\ 6 \\ 0 \\ -8 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 4 & 2 \\ -3 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 5 \\ -1 \\ 17 \\ -11 \\ 8 \end{pmatrix}$$

Problem 2 (Farkas Lemma): Prove Farkas' lemma using Gale's theorem.

Problem 3 (Helly's Theorem): Complete the proof of Helly's Theorem (Theorem 3.8).

Problem 4 (Minkowski-Weyl's Theorem for Polyhedra): Derive Theorem 3.9 from Theorem 3.10.

Problem 5 (Pointed Cone): Complete the proof of Corollary 3.13, namely prove that if P is a pointed cone $\{x : Ax \leq \mathbf{0}\}$, then there exists a vector c such that $c^T x > 0$ for all nonzero $x \in P$.

*Please hand in your solution to May Szedlák CAB G19.2 (may.szedlak@inf.ethz.ch) no later than **Tuesday, March 22, 2015**.