Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

# Polyhedral Computation, Spring 2016
# Assignment 1*

February 24, 2016

**Problem 1 (Rational Numbers):** For any two rational numbers $r$ and $s$, show that

- $\text{size}(r \times s) \leq \text{size}(r) + \text{size}(s)$
- $\text{size}(r + s) \leq 2\,(\text{size}(r) + \text{size}(s))$

**Problem 2 (Matrix Size):** To complete the proof of Theorem 2.4, derive $\text{size}(\hat{a}_{rs}) < 8\Delta$ when $r \leq k$.

**Problem 3 (Euclidean Algorithm):**

**(a)** Explain why the Euclidean algorithm is correct.

**(b)** Analyze the time complexity of the algorithm.

**Problem 4 (Hermite Normal Form):** Consider the integral matrix:

$$A = \begin{pmatrix} -4 & 6 & -6 & -6 \\ 6 & -3 & -9 & -3 \\ 4 & -3 & 9 & -3 \end{pmatrix}.$$

**(a)** Compute the Hermite normal form $[B, 0]$ of the matrix $A$ by hand.

**(b)** Solve the diophantine equation systems $Ax = b$ and $Ax = b'$ with $x \in \mathbb{Z}^4$ where

$$b = \begin{pmatrix} 0 \\ 12 \\ 18 \end{pmatrix}, \qquad b' = \begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix}.$$

If a system has no solution, find a certificate of infeasibility.

*Please hand in your solution to May Szedlák CAB G19.2 (may.szedlak@inf.ethz.ch) no later than **Tuesday, March 9, 2016, 15:00**.

**(c)** Compute the transformation matrix $T$ such that $A\,T = [B, 0]$ either by hand or any symbolic mathematics system (such as maple or mathematica).

**(d)** Find the general solution to any of the feasible systems in (b).

**Problem 5 (Lattice Basis):** Let $A \in \mathbb{Q}^{m\times n}$ be a rational matrix of full row rank, let $B$ be a basis of the lattice $L(A)$ and let $B'$ be a nonsingular $m \times m$ matrix whose column vectors are points in $L(A)$. Show that

- $|\det(B)| \leq |\det(B')|$.
- $B'$ is a basis of $L(A)$ if and only if $|\det(B)| = |\det(B')|$.