

QUALITATIVE MODELING AND SIMULATION: PROMISE OR ILLUSION

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ABSTRACT

In this panel discussion, questions shall be addressed relating to the potential usefulness of qualitative modeling and simulation. When might qualitative versus quantitative modeling/simulation be justified? What types of qualitative models if any are useful and under which conditions? What can qualitative simulations reveal that quantitative simulations cannot? Under what conditions are combined quantitative/qualitative models feasible/meaningful?

In this article, some basic definitions are presented that may serve as a basis for the discussion. After all, we must first agree on some common ground before particular properties can be explored.

1 TYPES OF MATHEMATICAL MODELS

What types of mathematical models do exist? A first category is the set of *continuous-time models*. Figure 1 shows how a state variable  $x$  changes over time in a continuous-time model.

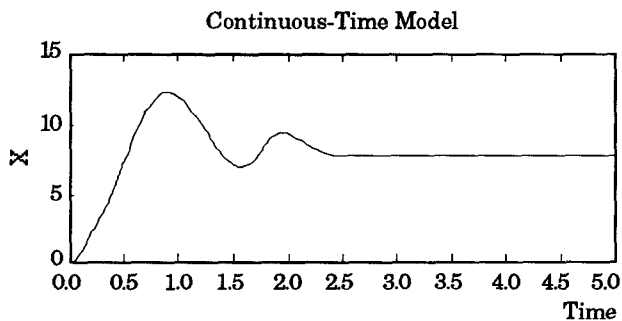


Figure 1: Trajectory behavior of continuous models

We can give the following definition for continuous-time models: "Continuous-time models are characterized by the fact that, within a finite time span, the

state variables change their values infinitely often." No other mathematical model shares this property.

Continuous-time models are represented through sets of differential equations. Among the continuous-time models, two separate classes can be distinguished: the *lumped parameter models*, which are described by *ordinary differential equations* (ODEs), in general:

$$\dot{x} = f(x, u, t) \tag{1}$$

and for the special case of *linear systems*:

$$\dot{x} = Ax + Bu \tag{2}$$

and the *distributed parameter models*, which are described by *partial differential equations* (PDEs) such as the diffusion equation:

$$\frac{\partial u}{\partial t} = \sigma \cdot \frac{\partial^2 u}{\partial x^2} \tag{3}$$

The second class of mathematical models to be mentioned is the set of *discrete-time models*. Figure 2 depicts the trajectory behavior exhibited by discrete-time models.

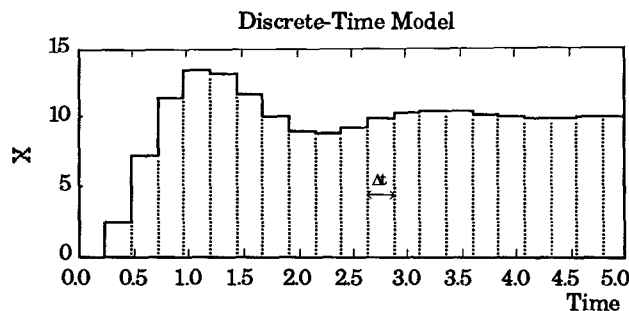


Figure 2: Trajectory behavior of discrete-time models

In this type of models, the time axis is discretized. Discrete-time models are commonly represented through sets of *difference equations*, at least if the discretization is equidistantly spaced. Such models can be represented as:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, t_k) \quad (4)$$

The third and final class of models is the set of *discrete-event models*. Paradoxically, the time axis of discrete-event models is usually “continuous” (i.e., *real* rather than *integer*), but discrete-event models differ from the continuous-time models by the fact that, in a finite time span, only a finite number of state changes may occur. Figure 3 depicts the typical trajectory behavior of a state variable in a discrete-event simulation.

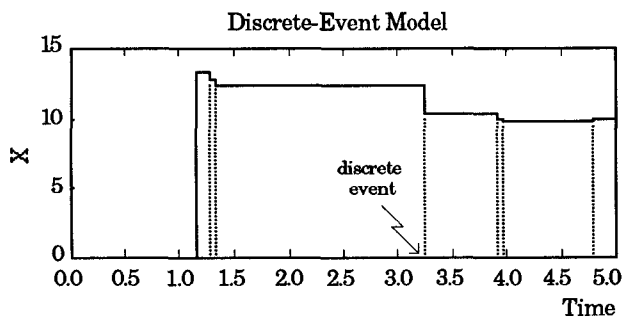


Figure 3: Trajectory behavior of discrete-event models

Discrete-event models are usually described by an enumeration of all possible event types together with either a list of times when these events occur (a so-called event calendar) or a set of conditions under which they occur (activity scanning).

The three types of models differ in their interpretation of *time*. In continuous-time simulations, time is (at least conceptually) an analog variable. Digital continuous-time simulations proceed by advancing the simulation clock in sufficiently small steps so that the human observer of the simulation results is seduced into smoothing out the trajectory behavior, i.e., he or she does not perceive the (on a digital computer necessary) time quantization as an essential or noteworthy property of his or her model. Discrete-time simulations perceive time as an integer or fixed point variable. Time is advanced by a fixed clock increment that is sufficiently large to make it essential/noteworthy. Discrete-event simulations finally proceed from one event time to the next, i.e., while time is perceived as a real variable, only par-

ticular time instants are seen as noteworthy, namely those where a state change occurs.

Notice that nothing is said about the nature of the state variables themselves. The three model/simulation types differ only in their interpretation of time. If the state variables themselves are real valued, the model is called *quantitative*, otherwise it is called *qualitative*. Figure 4 shows the trajectory behavior of a qualitative model.

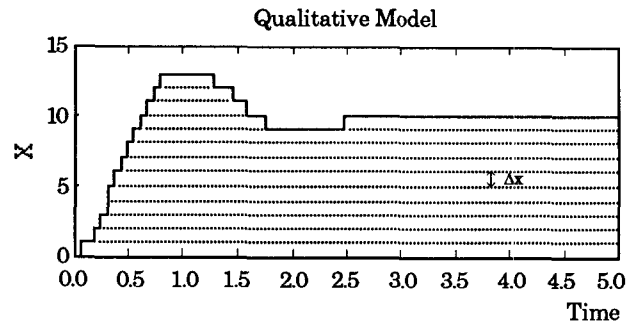


Figure 4: Trajectory behavior of qualitative models

Different types of qualitative models differ in the manner in which the state variables are discretized, and in the manner in which time is advanced.

## 2 SOME USEFUL DEFINITIONS

The following terminology shall be used:

1. *Qualitative variables* are variables that assume a finite ordered set of qualitative values, such as “minuscule,” “small,” “average,” “large,” and “gigantic.” The literature on quantitative soft sciences is a little more precise on this definition than the literature on artificial intelligence. For instance, Babbie (1989) distinguishes between:
  - (a) *Nominal measures*, i.e., variables whose values have the only characteristics of exhaustiveness and mutual exclusiveness. Nominal measures are unordered sets. Typical nominal variables might be the religious affiliation, or the hair color of a person. Such variables are not useful as state-variables in a simulation. They can play a role as parameters.
  - (b) *Ordinal measures*, i.e., variables that are nominal, and in addition, are rank-ordered. These variables are what I called above qualitative variables. However, sometimes

we shall let go of the condition of mutual exclusiveness, for example, when we operate on *fuzzy sets*.

- (c) *Interval measures*, i.e., variables that are ordinal, and in addition, have the property that a distance measure can be defined between any two values, that is: interval variables can be added to and/or subtracted from each other. A typical candidate for a “soft” interval variable might be the intelligence quotient.
  - (d) *Ratio measures*, i.e., variables that are interval measures, and in addition, have a true zero point.
2. *Qualitative behavior* denotes a time-ordered set of values of a qualitative variable, i.e., an episode. *Episodes* are qualitative trajectories.
  3. *Qualitative models* are models that operate on qualitative states.
  4. A *qualitative simulation* is an episode generator that infers qualitative behavior from a qualitative model.

Qualitative state variables are frequently ordinal measures, i.e., no distance information is preserved between neighboring states. In this respect, Fig.4 is atypical. Also, *time* in a qualitative simulation is often perceived as a qualitative variable as well. One “unit” of *qualitative time* is the time that elapses between two consecutive state changes. Obviously, since state changes don’t have to be equidistantly spaced, a “unit” of qualitative time is not a quantum. While qualitative models with quantitative time can be represented through integer-state discrete-event models, qualitative models with qualitative time could be represented through integer-state discrete-time models. However, a more commonly used representation for such models is the *finite state machine*.

While quantitative simulation always denotes the trajectory behavior of a quantitative model *in response to a particular experiment*, qualitative simulation often aspires to describe the episodic behavior of a qualitative model *in response to all possible experiments*. Consequently, while the result of a quantitative simulation is one particular trajectory behavior, the result of a qualitative simulation may be a (possibly extensive) set of all feasible episodic behaviors. It is desirable to describe a qualitative model in such a way as to minimize the cardinality of this set. However, this is often not possible, and it is therefore quite common that a qualitative simulation drowns in the sea of ambiguity. Are there techniques

around that minimize ambiguity in a systematic fashion? I don’t know of any such techniques. Hopefully, one of the panel members will address this important issue.

### 3 TYPES OF QUALITATIVE MODELS

#### 3.1 Naïve Physics Models

Naïve physics encompasses a set of different techniques for knowledge-based reasoning about physical systems. Different representatives of this type of qualitative models are described in Bobrow (1985). All naïve physics models have in common that they discretize the state space in a very crude way, namely the set  $\{- 0 +\}$ . A state variable is characterized as either *negative*,  $-$ , *zero*,  $0$ , or *positive*,  $+$ . Some researchers view these as three different states, while others view  $-$  and  $+$  as two different states with  $0$  being a “landmark” separating the two states. Both interpretations are meaningful. The former uses  $0$  to represent any state that is sufficiently small in magnitude, whereas the latter interprets  $0$  as a transitory condition that separates the states  $-$  and  $+$ .

The legitimation for this methodology is taken from the observation that, in a mechanical system, the direction of forces seems to carry more information than the magnitude. We don’t need to know the magnitude of the gravitational force to conclude that all free-moving objects will eventually fall towards the gravitational center.

Some researchers have observed that, while the magnitude of the state variables may still be important to qualitatively describe the behavior of a system, the magnitude of their time derivatives may no longer be important. Consequently, some researchers describe their models in terms of derivatives only (qualitative derivatives are sometimes referred to as *confluences*). Others extended the methodology by allowing additional levels and landmarks for the state variables while restricting state derivatives to the original ternary set.

Some researchers describe their naïve physics models through qualitative state equations, others through a rule base (a constraint set relating qualitative variables to each other). Morgan (1990) describes an automated procedure to translate a qualitative state-space model into a finite state machine (i.e., a special type of a rule base). Many of these researchers concentrate more on the modeling than on the simulation aspects, i.e., for them the generation of the qualitative model is more important than the generation of episodic behavior. Those that generate episodic behavior usually operate on qualitative

rather than quantitative time.

### 3.2 Inductive Reasoning Models

Inductive reasoning describes a variety of pattern-based techniques to reason about relations between qualitative variables. While naïve physics models strive to capture the *structure* of models, inductive reasoning models aspire to capture the *behavior* directly. Consequently, inductive reasoners generate directly and immediately finite state machine representations of systems, whereas naïve physics models arrive at such a representation only indirectly if at all. A good overview of various inductive reasoning approaches is given in Klir (1985).

Inductive reasoners are based on time quantization in addition to state quantization. Consequently, they operate on quantitative rather than qualitative time. Inductive reasoning models are *optimization models*. They optimize the forecasting power of the model by minimizing the indeterminism or ambiguity of state transitions. Cellier (1991) describes techniques for optimizing the state and time discretizations. However, it is equally important to select a representative set of state variables to base the model upon. I don't know of any technique that would allow us to systematically determine an optimal set of state variables. Hopefully, one of the panel members will address this important issue.

Contrary to the finite state representations generated from naïve physics models, inductive reasoners contain information about the likelihood of any particular state transition. This is important for model validation purposes. If the accumulated likelihood of a particular episode drops below a level that can be user-specified, forecasting will come to a halt. Therefore, the user can guarantee that his or her model will not forecast behavior beyond a time for which the available data are insufficient to substantiate the prediction.

Similar to the naïve physics models, it is possible to use inductive reasoners to enumerate all possible (or likely) system behaviors.

### 3.3 Symbolic Discrete-Event Models

As mentioned earlier, discrete-event models are equally well suited to represent qualitative as quantitative models. In the qualitative case, the event calendar must contain the time instants when the system transits from one qualitative state to another. However, as qualitative states may be fuzzy, also state transitions will be unsharp, and therefore also event times. Ambiguity can occur since a unique sequence of events may no longer be given. Due to the fuzziness

of event times, it could be that one event precedes another or vice-versa. Symbolic discrete-event models allow us to formulate event times as polynomials of *time* with unknown or partially known (possibly fuzzy) coefficients. Symbolic discrete-event simulation generates all trajectories that are feasible due to the fuzziness of these parameters. This technique has been described in Zeigler and Chi (1991). The fuzziness of event times is expressed through so-called time windows. The time window information is subsequently also used for the purpose of fault diagnosis.

Symbolic discrete-event models provide us with a second knowledge-based approach to reason about qualitatively known systems.

### 3.4 Neural Network Models

Neural networks provide us with a second pattern-based approach to qualitative (and even quantitative) modeling. Contrary to the inductive reasoners discussed earlier, the number of discrete states of a neural network can be quite large. Some neural networks (such as backpropagation networks) even operate on a continuous state-space, i.e., can be used for the identification of quantitative models. However, such models cannot be used to enumerate all possible system behaviors, and no information is generated as to the likelihood of a particular behavioral pattern. Yet, neural network models can be very powerful tools for capturing the behavior of (partially) unknown systems. An overview of this technique is given in Cellier (1991).

## 4 COMBINED QUANTITATIVE AND QUALITATIVE MODELS

It is quite common that properties of a system are partially known and partially unknown. It is therefore desirable if available knowledge about a system can be coded into the model while unknown properties can be treated in a qualitative manner. This justifies the call for combined quantitative and qualitative models. However, the generation of combined models is basically virgin territory. Very little has been written about such combined models. Obviously, the discrete-event approach lends itself naturally to such models. However, this approach requires that the model as a whole be formulated as a discrete-event model. This is not always desirable. Inductive reasoners could be integrated with continuous-time models using the fuzzy measure approach introduced by Li and Cellier (1990). However, no practical experience is available yet. It is not clear to me how naïve physics models could be combined with quantitative

models. I hope that one or the other of the panel members will address the issue of how which type of qualitative models can be combined with quantitative models.

## 5 CRITICAL ASSESSMENT

While there exists a good amount of literature already describing various methodologies for qualitative modeling (as described above), very few references discuss practical real-life applications of these techniques (applications that reach beyond simple schoolbook examples). It is not clear how well this technique scales up to solve realistically large problems. I hope that one or the other of the panel members will discuss practical experiences with qualitative modeling and simulation.

## REFERENCES

- Babbie, E. 1989. *The Practice of Social Research*, 5th edition, Wadsworth Publishing Company, Belmont, Calif.
- Bobrow, D.G. 1985. *Qualitative Reasoning About Physical Systems*, MIT Press, Cambridge, Mass.
- Cellier, F.E. 1991. *Continuous System Modeling*, Springer-Verlag, New York.
- Klir, G.J. 1985. *Architecture of Systems Problem Solving*, Plenum Press, New York.
- Li, D. and F.E. Cellier. 1990. "Fuzzy Measures in Inductive Reasoning," *Proceedings 1990 Winter Simulation Conference*, New Orleans, La., pp. 527-538.
- Morgan, A.J. 1990. "Accuracy in Qualitative Descriptions of Behaviour," *Proceedings 1990 Winter Simulation Conference*, New Orleans, La., pp. 520-526.
- Zeigler, B.P. and S.D. Chi. 1991. "Symbolic Discrete Event System Specification," *Proceedings IEEE Conference on AI, Simulation and Planning in High Autonomy Systems*, Cocoa Beach, Fla., pp. 130-141.

## AUTHOR BIOGRAPHY

**FRANCOIS E. CELLIER** received his B.S. degree in Electrical Engineering from the Swiss Federal Institute of Technology (ETH) Zurich in 1972, his M.S. degree in Automatic Control in 1973, and his Ph.D. degree in Technical Sciences in 1979, all from the same university. Following his Ph.D., Dr. Cellier worked as a Lecturer at ETH Zurich. He joined the University of Arizona in 1984 as an Associate Professor.

Dr. Cellier's main scientific interests concern modeling and simulation methodology, and the design of advanced software systems for simulation, computer-aided modeling, and computer-aided design. He has designed and implemented the GASP-V simulation package, and he was the designer of the COSY simulation language a modified version of which under the name of SYSMOD has meanwhile become a standard by the British Ministry of Defence. Dr. Cellier has authored or co-authored more than fifty technical publications, and he just published his first textbook on continuous system modeling. He served as a chairman of the National Organizing Committee (NOC) of the Simulation'75 conference, and as a chairman of the International Program Committee (IPC) of the Simulation'77 and Simulation'80 conferences, and he has also participated in several other NOC's and IPC's. He is associate editor of several simulation related journals, and he served as vice-chairman of two committees on standardization of simulation and modeling software.