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COMBINED QUALITATIVE/QUANTITATIVE SIMULATION MODELS OF CONTINOUS- TIME PROCESSES USING FUZZY INDUCTIVE REASONING TECHNIQUES

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Abstract. The feasibility of mixed quantitative and qualitative simulation is demonstrated by means of a simple hydraulic control system. The mechanical and electrical parts of the control system are modeled using differential equations, whereas the hydraulic part is modeled using fuzzy inductive reasoning. The mixed quantitative and qualitative model is simulated in ACSL, and the simulation results are compared with those obtained from a fully quantitative model. The example was chosen as a simple to describe, yet numerically demanding process whose sole purpose is to prove the concept. Several practical applications of this mixed modeling technique are mentioned in the paper, but their realization has not yet been completed.

Keywords. Modeling; simulation; mixed quantitative and qualitative models; inductive reasoning; forecasting theory; fuzzy systems; learning systems; artificial intelligence.

INTRODUCTION

Qualitative simulation has recently become a fashionable branch of research in artificial intelligence. Human reasoning has been understood as a process of mental simulation, and qualitative simulation has been introduced as an attempt to replicate, in the computer, facets of human reasoning.

Qualitative simulation can be defined as evaluating the behavior of a system in qualitative terms (Cellier, 1991b). To this end, the states that the system can be in are lumped together to a finite (discrete) set. For example, instead of dealing with temperature as a real–valued quantity with values such as 22.0°C, or 71.6°F, or 295.15 K, qualitative temperature values may be characterized as 'cold,' 'warm,' or 'hot.'

Qualitative variables are variables that assume qualitative values. Variables of a dynamical system are functions of time. The behavior of a dynamical system is a description of the values of its variables over time. The behavior of quantitative variables is usually referred to as trajectory behavior, whereas the behavior of qualitative variables is commonly referred to as episodical behavior. Qualitative simulation can thus be defined as the process of inferring the episodical behavior of a qualitative dynamical system or model.

Qualitative variables are frequently interpreted as an ordered set without distance measure (Babbie, 1989). It is correct that 'warm' is "larger" (warmer) than 'cold,' and that 'hot' is "larger" (warmer) than 'warm.' Yet, it is not true that

$$\text{'warm'} - \text{'cold'} = \text{'hot'} - \text{'warm'} \qquad (1)$$

or, even more absurd, that

$$\text{'hot'} = 2 * \text{'warm'} - \text{'cold'} \qquad (2)$$

No '−' operator is defined for qualitative variables.

Time, in a qualitative simulation, is also frequently treated as a qualitative variable. It is then possible to determine whether one event happens before or after another event, but it is not possible to specify when precisely a particular event takes place.

The most widely advocated among the qualitative simulation techniques are the knowledge–based approaches that were originally derived from the *Naïve Physics Manifesto* (Hayes, 1979). Several dialects of these types of qualitative models exist (de Kleer and Brown, 1984; Forbus, 1984; Kuipers, 1986). They are best summarized in (Bobrow, 1985).

The purpose of most qualitative simulation attempts is to enumerate, in qualitative terms, all possible episodical behaviors of a given system under all feasible experimental conditions. This is in direct contrast to quantitative simulations that usually content themselves with generating one single trajectory behavior of a given system under one single set of experimental conditions.

MIXED MODELS

In the light of what has been explained above, it seems questionable whether mixed quantitative and qualitative models are feasible at all. How should a mixed quantitative and qualitative simulation deal with the fact that the quantitative subsystems treat the independent variable, *time*, as a quantitative variable, whereas the qualitative subsystems treat the same variable qualitatively? When does a particular qualitative event occur in terms of quantitative time? How are the explicit experimental conditions that are needed by the quantitative subsystems accounted for in the qualitative subsystems?

Quite obviously, a number of incompatibility issues exist between quantitative and qualitative subsystems that must be settled before mixed simulations can be attempted. In a mixed simulation, also the qualitative subsystems must treat time as a quantitative variable. Furthermore, the purpose of qualitative models in the context of mixed simulations is revised. It is no longer their aim to enumerate episodical behaviors. Instead, also the qualitative models are now used to determine a single episodical behavior in response to a single set of qualitative experimental conditions.

Do so revised qualitative models make sense? It is certainly illegitimate to request that, because human pilots are unable to solve Riccati equations in their heads to determine an optimal flight path, autopilots shouldn't tackle this problem either. It is not sufficient to justify the existence of qualitative models by human inadequacies to deal with quantitative information.

Two good reasons for dealing with information in qualitative ways are the following:

1. Quantitative details about a (sub)system may not be available. For example, while the mechanical properties of a human heart are well understood and can easily be described by differential equation models, the effects of many chemical substances on the behavior of the heart are poorly understood and cannot easily be quantified. A mixed model could be used to describe those portions of the overall system that are well understood by quantitative differential equation models, while other aspects that are less well understood may still be representable in qualitative terms.
2. Quantitative details may limit the robustness of a (sub)system to react to previously unknown experimental conditions. For example, while a human pilot is unable to compute an optimal flight path, he or she can control the airplane in a much more robust fashion than any of today's autopilots. Optimality in behavior can be traded for robustness. A fuzzy controller is an example of a qualitative subsystem that is designed to deal with a larger class of experimental conditions in suboptimal ways.

Mixed quantitative and qualitative models may be used to address either or both of the above applications. However, in order to do so, it is necessary to devise qualitative modeling and simulation capabilities that are compatible with their quantitative counterparts and that can be used to represent qualitative subsystems as those mentioned above appropriately and in terms of knowledge available to the system designer at the time of modeling.

It is the purpose of this paper to describe one such mixed modeling and simulation methodology. In the advocated approach, the qualitative subsystems are represented (modeled) by a special class of finite state machines called fuzzy optimal masks, and their episodical behavior is inferred (simulated) by a technique called fuzzy forecasting. The overall process of qualitative modeling and simulation is referred to as fuzzy inductive reasoning.

FUZZY RECODING

Recoding denotes the process of converting a quantitative variable to a qualitative variable. In general, some information is lost in the process of recoding. Obviously, a temperature value of 97°F contains more information than the value 'hot.' Fuzzy recoding avoids this problem. Figure 1 shows the fuzzy recoding of a variable called "systolic blood pressure."

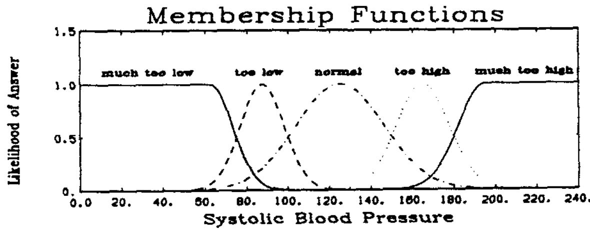

Figure 1. Fuzzy recoding.

For example, a quantitative systolic blood pressure of 135.0 is recoded into a qualitative value of 'normal' with a fuzzy membership function of 0.895 and a side function of 'right.' Thus, a single quantitative value is recoded into a triple. Any systolic blood pressure with a quantitative value between 100.0 and 150.0 will be recoded into the qualitative value 'normal.' The fuzzy membership function denotes the value of the bell–shaped curve shown on Fig.1, always a value between 0.5 and 1.0, and the side function indicates whether the quantitative value is to the left or to the right of the maximum of the fuzzy membership function. Obviously, the qualitative triple contains the same information as the original quantitative variable. The quantitative value can be regenerated accurately from the qualitative triple, i.e., without any loss of information.

Due to space limitations, details of how quantitative variables are optimally recoded into qualitative triples will not be given in this paper. These details are provided in (Li and Cellier, 1990; Cellier, 1991a).

FUZZY OPTIMAL MASKS

A mask denotes relationship between different variables. For example, given the following raw data model consisting of five variables, namely the inputs u_1 and u_2 and the outputs y_1, y_2, and y_3 that are recorded at different values of time.

$$\begin{array}{l} \textit{time} \\ 0.0 \\ \delta t \\ 2\cdot\delta t \\ 3\cdot\delta t \\ \vdots \\ (n_{rec}-1)\cdot\delta t \end{array} \quad \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \end{array} \tag{3}$$

Each column of the raw data model contains one qualitative variable recorded at different values of time, and each row contains the recordings of all qualitative variables at one point in time. The raw data matrix is accompanied by a fuzzy membership matrix and a side matrix of the same dimensions.

A mask denotes a relationship between these variables. For example, the mask

$$\begin{array}{l} {}_t\backslash^x \\ t-2\delta t \\ t-\delta t \\ t \end{array} \quad \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & -2 & -3 & 0 & 0 \\ -4 & 0 & +1 & 0 & 0 \end{pmatrix} \end{array} \tag{4}$$

denotes the following relationship pertaining to the five variable system

$$y_1(t) = \tilde{f}(y_3(t-2\delta t), u_2(t-\delta t), y_1(t-\delta t), u_1(t)) \tag{5}$$

Negative elements in the mask matrix denote inputs of the qualitative functional relationship. The example mask has four inputs. The sequence in which they are enumerated is immaterial. They are usually enumerated from left to right and top to bottom. A positive element in the mask matrix denotes the output. Thus, Eq.(4) is simply a matrix representation of Eq.(5). The mask must have the same number of columns as the raw data matrix. The number of rows of the mask matrix is called the *depth* of the mask. The mask can be used to flatten a dynamic relationship out into a static relationship. The mask can be shifted over the episodical behavior. Selected inputs and outputs can be read out from the raw data matrix and can be written on one row next to each other. Figure 2 illustrates this process.

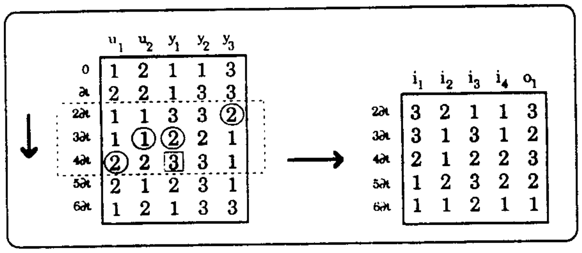

Figure 2. Flattening dynamic relationships through masking.

After the mask has been applied to the raw data, the formerly dynamic episodical behavior has become static, i.e., the relationship is now contained within a single row

$$o_1(t) = \tilde{f}(i_1(t), i_2(t), i_3(t), i_4(t)) \tag{6}$$

The resulting matrix is called *input/output matrix.*

How is the mask selected? A *mask candidate matrix* is constructed in which negative elements denote potential inputs, and the single positive element denotes the true output of the mask. A good mask candidate matrix for the previously mentioned five variable system might be

$$\begin{array}{c} {}_t\backslash^x \\ t-2\delta t \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & 0 & 0 \end{pmatrix} \end{array} \tag{7}$$

A mask candidate matrix is an ensemble of all acceptable masks. The optimal mask selection algorithm determines the best among all masks that are compatible with the mask candidate matrix. The mask of Eq.(4) is one such mask. The optimal mask is the one mask that maximizes the forecasting power of the inductive reasoning process, i.e., the mask that results in the most deterministic input/output matrix.

Due to space limitations, the details of how the optimal mask selection algorithm works are omitted from this paper. These details are also provided in (Li and Cellier, 1990; Cellier, 1991a).

FUZZY FORECASTING

Once the optimal mask has been determined, it can be applied to the given raw data matrix resulting in a particular input/output matrix. Since the input/output matrix contains functional relationships within single rows, the rows of the input/output matrix can now be sorted in alphanumerical order. The result of this operation is called the *behavior matrix* of this system. The behavior matrix is a finite state machine. For each combination of input values, it shows which output is most likely to be observed.

Forecasting is now a straightforward procedure. The mask is simply shifted further down beyond the end of the raw data matrix, future inputs are read out from the mask, and the behavior matrix is used to determine the future output, which can then be copied back into the raw data matrix. In fuzzy forecasting, it is essential that, together with the qualitative output, also a fuzzy membership value and a side value are forecast. Thus, fuzzy forecasting predicts an entire qualitative triple from which a quantitative variable can be regenerated whenever needed.

In fuzzy forecasting, the membership and side functions of the new input are compared with those of all previous recordings of the same qualitative input contained in the behavior matrix. The one input with the most similar membership and side functions is identified. For this purpose, a cheap approximation of the regenerated quantitative signal

$$d = 1 + side * (1 - Memb) \tag{8}$$

is computed for every input variable of the new input set, and the regenerated d_i values are stored in a vector. This reconstruction is then repeated for all previous recordings of the same input set. Finally, the $\mathcal{L}_2$ norms of the difference between the d vector of the new input and the d vectors of all previous recordings of the same input are computed, and the previous recording with the smallest $\mathcal{L}_2$ norm is identified. Its *output* and *side* values are then used as forecasts for the *output* and *side* values of the current state.

Forecasting of the new membership function is done a little differently. Here, the five previous recordings with the smallest $\mathcal{L}_2$ norms are used (if at least five such recordings are found in the behavior matrix), and a distance–weighted average of their fuzzy membership functions is computed and used as the forecast for the fuzzy membership function of the current state.

More details of fuzzy forecasting are provided in (Cellier, 1991a).

AN EXAMPLE

In the remainder of this paper, an example will be presented that demonstrates, for the first time, the process of mixed quantitative and qualitative simulation using fuzzy inductive reasoning. The example was chosen simple enough to be presented in full, yet complex enough to demonstrate the generality and validity of the approach. However, it is not suggested that the chosen example represents a meaningful application of mixed quantitative and qualitative simulation. The example was chosen to prove the concept and to clearly illustrate the procedure, not as a realistic application of the proposed technique.

Figure 3 shows a hydraulic motor with a four–way servo valve.

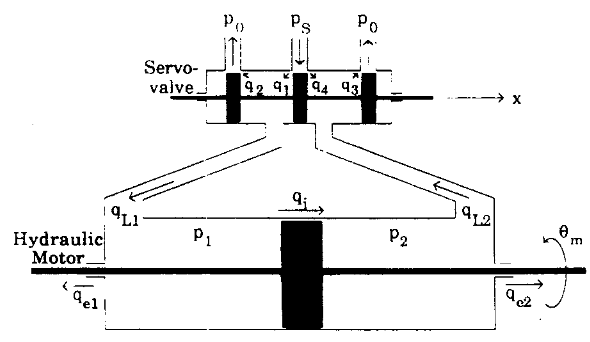

Figure 3. Hydraulic motor with a four–way servo valve.

The flows from the high–pressure line into the servo valve and from the servo valve back into the low–pressure line are turbulent. Consequently, the relation between flow and pressure is quadratic

$$q_1 = k(x_0 + x)\sqrt{P_S - p_1} \tag{9a}$$
$$q_2 = k(x_0 - x)\sqrt{p_1 - P_0} \tag{9b}$$
$$q_3 = k(x_0 + x)\sqrt{p_2 - P_0} \tag{9c}$$
$$q_4 = k(x_0 - x)\sqrt{P_S - p_2} \tag{9d}$$

The change in the chamber pressures is proportional to the effective flows in the two chambers

$$\dot{p}_1 = c_1(q_{L1} - q_i - q_{e1} - q_{ind}) \tag{10a}$$
$$\dot{p}_2 = c_1(q_{ind} + q_i - q_{e2} - q_{L2}) \tag{10b}$$

where the internal leakage flow, q_i, and the external leakage flows, q_{e1} and q_{e2}, can be computed as

$$q_i = c_i \cdot p_L = c_i(p_1 - p_2) \tag{11a}$$
$$q_{e1} = c_e \cdot p_1 \tag{11b}$$
$$q_{e2} = c_e \cdot p_2 \tag{11c}$$

The induced voltage, q_{ind}, is proportional to the angular velocity of the hydraulic motor, ω_m

$$q_{ind} = \psi \cdot \omega_m \tag{12}$$

and the torque produced by the hydraulic motor is proportional to the load pressure, p_L

$$T_m = \psi \cdot p_L = \psi(p_1 - p_2) \tag{13}$$

The hydraulic motor is embedded in the control circuitry shown on Fig.4

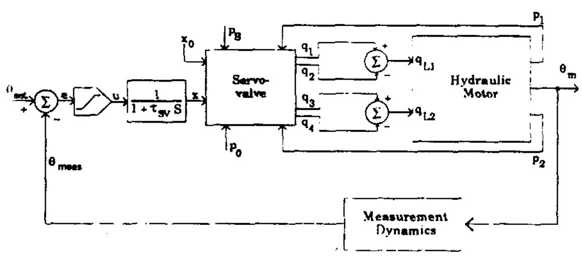

Figure 4. Hydraulic motor position control ciruit.

An ACSL program (MGA, 1985) was written that simulates the control system over 2.5 seconds. A binary random input signal was applied to the input of the system, θ_{set}, and the values of the the control signal, u, the angular velocity, $\omega_m = \dot{\theta}_m$, and the torque, T_m, of the hydraulic motor were recorded for later reuse.

For demonstration purposes, it is now assumed that no knowledge exists that would permit a description of the hydraulic equations by means of a differential equation model. All that is known is that the mechanical torque, T_m, of the hydraulic motor somehow depends on the control signal, u, and the angular velocity, ω_m.

In a mixed quantitative and qualitative simulation, the mechanical and electrical parts of the control system will be represented by differential equation models, whereas the hydraulic part will be represented by a fuzzy inductive reasoning model. The mixed simulation results will be compared with the previously obtained purely quantitative simulation results for validation purposes.

Optimal recoding would suggest that the three variables u, ω_m, and T_m be sampled once every 0.025 seconds if a mask depth of 3 is chosen. This value is deduced from the slowest time constant (eigenvalue of the Jacobian) to be covered by the mask. A more detailed explanation is provided in (Li and Cellier, 1990; Cellier, 1991a). Unfortunately, fuzzy inductive forecasting will predict only one value of T_m per sampling interval. Thus, the overall control system will react like a sampled–data control system with a sampling rate of 0.025. Thereby, the stability of the control system is lost. From a control system perspective, it is necessary to sample the variables considerably faster, namely once every 0.0025 seconds.

Therefore, it was decided to choose the following mask candidate matrix

$$\begin{array}{l} {}_t\backslash^x \\ t-20\delta t \\ t-19\delta t \\ \vdots \\ t-11\delta t \\ t-10\delta t \\ t-9\delta t \\ \vdots \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccc} u & \omega_m & T_m \end{array} \\ \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -1 & -1 & +1 \end{pmatrix} \end{array} \tag{14}$$

of depth 21. As mandated by control theory, the sampling interval δt is chosen to be 0.0025 seconds. Yet, as dictated by the inductive reasoning technique, the mechanical torque, T_m, at time t will depend on past values of u, ω_m, and T_m at times $t - 0.025$ and $t - 0.05$.

The optimal mask found with this mask candidate matrix is

$$\begin{array}{l} {}_t\backslash^x \\ t-20\delta t \\ t-19\delta t \\ \vdots \\ t-11\delta t \\ t-10\delta t \\ t-9\delta t \\ \vdots \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccc} u & \omega_m & T_m \end{array} \\ \begin{pmatrix} 0 & -1 & -2 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -3 & 0 & +1 \end{pmatrix} \end{array} \tag{15}$$

In other words

$$T_m(t) = \tilde{f}(\omega_m(t - 0.05), T_m(t - 0.05), u(t)) \tag{16}$$

The first 900 rows of the raw data matrix were used as past history data to compute the optimal mask. Fuzzy forecasting was used to predict new qualitative triples for T_m for the last 100 rows of the raw data matrix. From the predicted qualitative triples, quantitative values were then regenerated. Figure 5 compares the true "measured" values of T_m obtained from the purely quantitative simulation (solid line) with the forecast and regenerated values obtained from fuzzy inductive reasoning (dashed line).

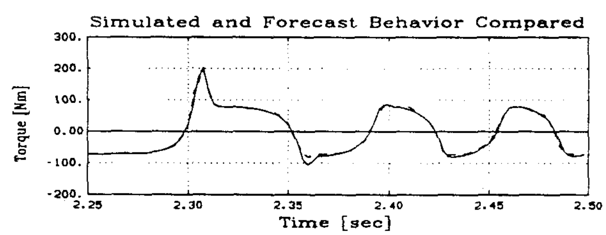

Figure 5. Simulated and forecast torque trajectories compared.

The results are encouraging. Quite obviously, the optimal mask contains sufficient information about the behavior of the hydraulic subsystem to be used as a valid replacement of the true quantitative differential equation model. Notice that the fuzzy inductive reasoning model was constructed solely on the basis of measurement data. No insight into the functioning of the hydraulic subsystem was required other than the knowledge that the torque, T_m, dynamically depends on the control signal, u, and the angular velocity, ω_m.

In a mixed quantitative and qualitative simulation, the fuzzy inductive reasoning model was then used to replace the former differential equation model of the hydraulic subsystem while the electrical and mechanical subsystems were described using differential equations as before. The mixed model is shown on Fig.6.

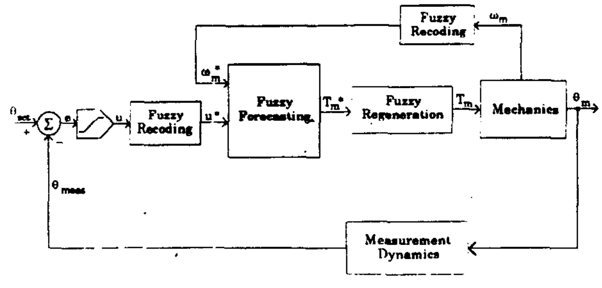

Figure 6. Mixed model of the hydraulic system.

The quantitative control signal, u, is converted to a qualitative triple, u^*, using fuzzy recoding. Also the quantitative angular velocity, ω_m, of the hydraulic motor is converted to a qualitative triple, ω_m^*. From these two qualitative signals, a qualitative triple of the torque of the hydraulic motor, T_m^*, is computed by means of fuzzy forecasting. This qualitative signal is then converted back to a quantitative signal, T_m using fuzzy signal regeneration. The mechanical parts of the hydraulic motor are simulated by means of a differential equation model. The same holds true for the measurement dynamics.

Forecasting was restricted to the last 100 sampling intervals, i.e., to the time span from 2.25 seconds to 2.5 seconds. Figure 7 compares the angular position, θ_m, of the hydraulic motor from the purely quantitative simulation (solid line) with that of the mixed quantitative and qualitative simulation (dashed line).

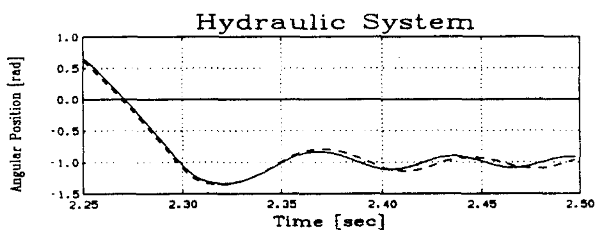

Figure 7. Comparison of quantitative and mixed simulations.

As was to be expected, the mixed model behaves like a sampled–data control system. The mixed simulation exhibits an oscillation amplitude that is slightly larger and an oscillation frequency that is slightly smaller than those shown by the purely quantitative simulation. Surprisingly, the damping of the mixed model is slightly larger than that of the purely quantitative model.

CONCLUSIONS

The example demonstrates the validity of the chosen approach. Mixed simulations are similar in effect to sampled–data system simulations. *Fuzzy recoding* takes the place of analog–to–digital converters, and *fuzzy signal regeneration* takes the place of digital–to–analog converters. However, this is where the similarity ends. Sampled–data systems operate on a fairly accurate representation of the digital signals. Typical converters are 12–bit converters, corresponding to discretized signals with 4096 discrete levels. In contrast, the fuzzy inductive reasoning model employed in the above example recoded all three variables into qualitative variables with the three levels 'small,' 'medium,' and 'large.' The quantitative information is retained in the fuzzy membership functions that accompany the qualitative signals. Due to the small number of discrete levels, the resulting finite state machine is extremely simple. Fuzzy membership forecasting has been shown to be very effective in inferring quantitative information about the system under investigation in qualitative terms.

Due to the space limitations inherent in a publication in conference proceedings it was not possible to provide, in this paper, any details of the programs used for simulation. Fuzzy inductive reasoning is accomplished using SAPS–II (Cellier, 1987), a software that evolved from the General System Problem Solving (GSPS) framework (Klir, 1985, 1989; Uyttenhove, 1979). SAPS–II is implemented as a (FORTRAN–coded) function library of CTRL–C (SCT, 1985). A subset of the SAPS–II modules, namely the recoding, forecasting, and regeneration modules have also been made available as an application library of ACSL (MGA, 1986), which is the software used in the mixed quantitative and qualitative simulation runs. More details will be provided in an enhanced version of this paper that is currently being prepared for submission to a journal.

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