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THE CAUSALITY HORIZON: LIMITATIONS TO PREDICTABILITY OF BEHAVIOR USING FUZZY INDUCTIVE REASONING

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ABSTRACT

This paper focuses on limitations to predictability of system behavior. The concept of a *causality horizon* is introduced that helps determine the likelihood of success of a qualitative prediction. The Fuzzy Inductive Reasoning (FIR) methodology is the tool used in this paper to model and simulate (forecast) the system behavior.

Two systems, a linear state-space model and a biomedical system, serve to demonstrate the concept. These very different types of systems were chosen in order to illustrate the differences in predictability between applications from technical domains and those from soft sciences areas. It is much more difficult to obtain decent predictions for soft science systems than to obtain accurate predictions for systems from the hard sciences. The purpose of this paper is to explain this discrepancy.

INTRODUCTION

Qualitative methodologies have been applied to a wide range of physical domains such as nuclear power plants, aerospace, and robotics, but also to soft sciences such as biomedicine, economy, and psychology. In all these cases, qualitative methods have been introduced as tools to form models of dynamic systems without precise knowledge of the underlying laws that govern the behavior of these systems and to predict, in qualitative terms, how they react to input stimuli.

The paper focuses on limitations to predictability of system behavior through induction. These limitations are demonstrated by applying a particular inductive modeling and simulation technique, called fuzzy inductive reasoning (FIR), to two types of systems: a linear state-space model, and observations of input/output behavior of a biomedical system. The *causality horizon* is introduced, a conceptual barrier limiting the predictability of future states of the system under investigation.

The inductive reasoning methodology was originally developed by G. Klir (Klir 1985) as a tool for general system analysis to study the conceptual modes of behavior of systems. One implementation of this methodology is SAPS-II (Cellier and Yandell 1987). Fuzzy measures were introduced into the methodology independently by (Klir and Folger 1988; Klir 1989; Wang and Klir 1992), and by (Li and Cellier 1990). Even more recently, SAPS-II has been propagated as a tool for qualitatively studying the behavior of highly complex non-linear technical systems (Cellier et al. 1992; de Albornoz and Cellier 1993a, 1993b) as well as biomedical systems (Nebot et al. 1993).

Experiences with these applications have shown that the quality of predictions is not always the same. In particular, it was much more difficult to obtain even half-way decent predictions for the biomedical application, whereas the predictions in the technical applications were accurate far beyond our original expectations. It is the purpose of this paper to illuminate and explain this discrepancy.

FUZZY INDUCTIVE REASONING

Identification of a FIR Model

For identifying a FIR model, a large set of rich data is required. The term "rich" refers to the quality of the data. The available data should reflect the possible behavioral patterns of the system to be modeled. The FIR model can smoothly *interpolate* between similar behavioral patterns, but it cannot *extrapolate* far beyond the horizon of previous experiences.

In SAPS-II, the knowledge about the system is represented by a large data matrix with one row per sample, and as many columns as there are variables in the system to be modeled. Thus, each row represents one data record, and each column represents one trajectory. This data matrix is called the *raw data matrix*. For example:

$$\begin{array}{l} \textit{time} \\ 0.0 \\ \delta t \\ 2\cdot\delta t \\ 3\cdot\delta t \\ \vdots \\ (n_{rec}-1)\cdot\delta t \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & y_1 & y_2 \end{array} \\ \begin{pmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \end{array} \quad (1)$$

where u_i are the inputs, y_i the outputs, n_{rec} is the number of data records, and δt is the sampling interval.

From the raw data matrix, the FIR methodology is able to identify a model of a given system for the purpose of fore-

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casting its future behavior for any given input stream that does not lead to system behavior much outside the range of previously observed behavioral patterns. A model describes relationships between system variables. In the FIR methodology, these relationships are represented by a so–called *mask*. For instance, the following mask:

$$\begin{array}{c} {}_t\backslash^x \\ t-2\delta t \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & y_1 & y_2 \end{array} \\ \begin{pmatrix} 0 & 0 & -1 & 0 & -2 \\ 0 & -3 & 0 & 0 & 0 \\ -4 & 0 & 0 & +1 & 0 \end{pmatrix} \end{array} \quad (2)$$

represents the equation:

$$y_1(t) = \tilde{f}(u_3(t-2\delta t), y_2(t-2\delta t), u_2(t-\delta t), u_1(t)) \quad (3)$$

where $\tilde{f}$ denotes a qualitative functional relationship. The negative elements in the mask matrix denote inputs of the qualitative functional relationship. The sequence in which they are enumerated is immaterial. The positive value denotes the single output of the qualitative functional relationship. A mask has the same number of columns as the raw data matrix to which it should be applied, and it has a certain number of rows called the *depth* of the mask. Neighboring rows of the mask represent neighboring data records. Thus, a mask of depth n covers a time interval of $\Delta t = (n-1)\cdot\delta t$ time units.

In the process of qualitative modeling, each of a set of possible masks is compared to the others with respect to its potential merit. The optimality of the mask is evaluated with respect to the maximization of its forcasting power.

Determination of the Mask Depth

How should the time distance between two logged entries of the trajectory behavior, δt, and the time span to be covered by the mask, Δt, be chosen? Experience has shown that the mask should cover the largest time constant, t_l, and that the sampling rate, δt, should be no larger than half the shortest time constant, t_s, of the system to be captured by the model, thus:

$$\Delta t \geq t_l \quad ; \quad \delta t \leq \frac{t_s}{2} \quad (4)$$

The *depth* of the mask can then be computed as follows:

$$depth = \text{round}(\frac{\Delta t}{\delta t}) + 1 \quad (5)$$

How are the two time constants determined in practice? The concept of a time constant is borrowed from *linear* system theory. In a non–linear system, time constants can only be defined through the eigenvalues of its Jacobian, and may therefore be time–variant themselves. If the time constants vary too much over time, the FIR methodology may not be able to use one and the same mask for the entire time period to be explored. If an analytical (quantitative) model of the system under investigation is available, the relevant time constants can be read out from it. If at least the physical system itself is available for experimentation, a frequency response (Bode diagram) can be measured, and the time constants can be estimated from it. If only time series are available that may have been measured earlier by someone else, or if the physical system is not open to free experimentation, such as in the case of a biomedical system involving humans, spectra of the input and output signals can be determined, but the information obtained from those may be deceiving. Ultimately, the modeler may have to rely on expert opinion as to what these time constants may be.

In this paper, it is shown that the mask depth is not only dictated by the two time constants mentioned earlier, but is limited also by yet another factor that has been coined the *causality horizon.* Up to this point, no measure of the causality between inputs and outputs was taken into account. Seemingly, any two signals can be declared as "inputs" and "outputs" of a "system," and a system response can be predicted between them. Obviously, this cannot be done. A *measure of causality* should be introduced that allows to determine the likelihood of success of a qualitative prediction. Such a measure is the correlation function.

In the previously used mask, the output y_1 depended on current and past values of the inputs u_i and on past values of the outputs y_i. The autocorrelation function for y_1, as well as the crosscorrelation functions between u_i and y_1 and the crosscorrelation between y_2 and y_1 can be computed. All these functions decay for sufficiently large values of the time lapse Δt. The correlation functions can be viewed as measures of causality. Once a sufficiently long time span Δt has elapsed, the output, $y_1(t)$, is no longer causally related to any of the inputs, $u_i(t-\Delta t)$, the other output, $y_2(t-\Delta t)$, or its own past, $y_1(t-\Delta t)$, since the corresponding correlation functions for this value of Δt are small.

By making Δt larger and larger, the inductive reasoner is told to predict the future from old data values that are no longer causally related to the current time. This obviously can't work. The effect is that, even if the best possible mask spanning Δt time units is used, recurrences of the same input patterns lead to all legal output values with approximately equal probability. This is just another way of saying that the output does not causally depend on these inputs. The forecasting algorithm within the FIR methodology is therefore uncertain which value to predict and chooses one of the values arbitrarily, assigning to its forecast a low confidence value. In those cases, the forecast is poor and looks like noise.

If Δt is chosen smaller than the shortest time constant to be captured, the FIR forecast basically consists of a constant value (in lack of better knowledge, tomorrow's weather is predicted to be the same as today's). If Δt is increased to cover the fast time constants but not the slow ones, the forecast exhibits local maxima and minima where the real data show them, but the forecast won't follow the general trend, i.e., it cannot follow the slow time constants. If Δt is chosen sufficiently large for all time constants to be covered but not larger than the causality horizon, the forecast will be the best that can be obtained. All these types of behaviors can be seen in the following examples.

LINEAR SYSTEM

The linear system used in this example is described by the following equations:

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{pmatrix} * x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} * u \quad (6)$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} * x \quad (7)$$

It is demonstrated with this example, how the selections of δt and Δt influence the quality of the forecast. To this end, Δt is varied, and an optimal mask is computed in each case using a subset of the available data. These optimal masks are then used to forecast the remainder of the data stream.

Three major types of behavior can be observed in this experiment:

$\delta t < t_s, t_l < \Delta t < C_h$

In this experiment, δt and Δt were calculated using the characteristics of the linear system. This system exhibits two time constants, a slow (large) one of $t_l = 2.7$ seconds, and a fast (small) one of $t_s = 0.3$ seconds. Consequently, $\delta t = 0.15$ seconds and $\Delta t = 2.7$ seconds were used, which yields a mask depth of $depth = 19$.

With these values for Δt and δt, the reasoner operates in a region where both time constants are captured and, as shown in figure 1, the crosscorrelation function between input and output and the autocorrelation of the output are comfortably large. Therefore, the predictions of future output values are expected to be good, as can be verified in figure 2.

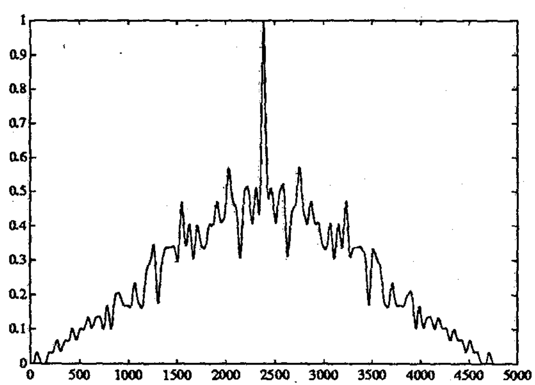

Figure 1: Crosscorrelation Function

$\delta t < t_s, t_l, C_h < \Delta t$

In this test, the concept of the causality horizon is illustrated. The mask depth is being repetitively increased, leading to a progressive deterioration of the prediction quality. This happens because the mask now stretches beyond the limit imposed by the causality horizon. When the mask depth is increased to a value of 101 rows, corresponding to $\Delta t = 15$ seconds, the prediction of future outputs looks like noise, as shown in figure 3. At that point, the crosscorrelation and autocorrelation functions have already decayed to approximately 60% of their maximum values, and the causal relation between input and the output is no longer sufficiently strong.

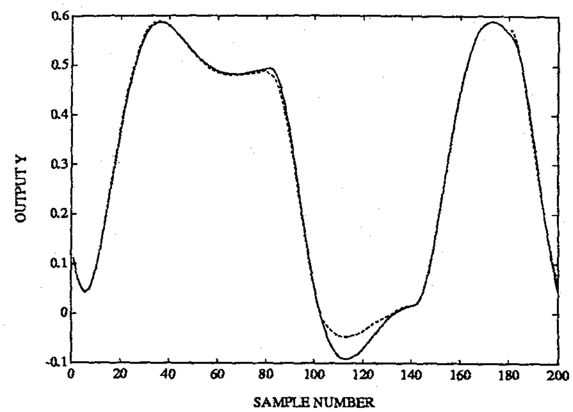

Figure 2: Real vs. Pted. Behavior (Mask Depth 19)

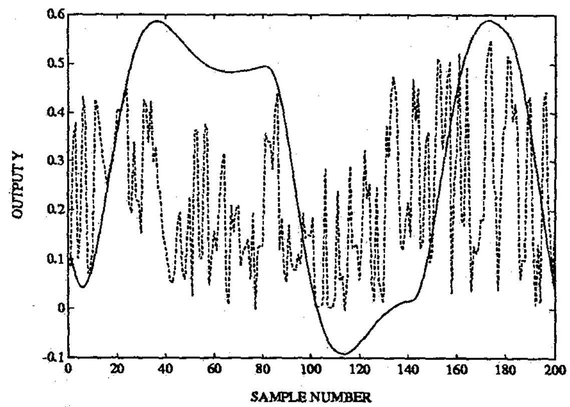

Figure 3: Real vs. Pted. Behavior (Mask Depth 101)

$\delta t, \Delta t < t_s, t_l, C_h$

Now, the time span covered by the mask is made smaller than the shortest time constant to be captured by the model. A δt of 0.003 seconds and a mask depth of $depth = 3$ are used in this test, therefore, Δt captures 0.006 seconds, a value smaller than the faster of the two time constants.

As was to be expected, the reasoner predicts a constant value for the output. This is shown in figure 4.

This example shows clearly that the upper limit of the predictability of system behavior is defined by the causality horizon. The correlation functions provide a causality measure that is essential for understanding why the quality of predictions is not always the same. Figure 5 shows the *forecast quality*, measured as:

$$Q = 1 - \frac{\varepsilon_{\mathrm{RMS}}}{\bar{y}} \quad (8)$$

where $\varepsilon_{\mathrm{RMS}}$ denotes the sqare root of the mean value of the squared error over the prediction period, and $\bar{y}$ denotes the

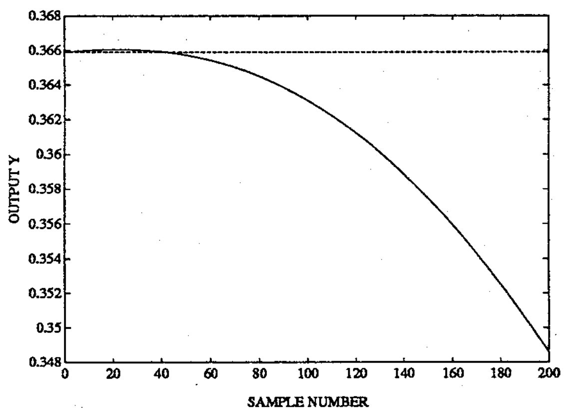

Figure 4: Real vs. Pted. Behavior (Mask Depth 3)

mean value of the predicted output over the same period plotted across the mask depth.

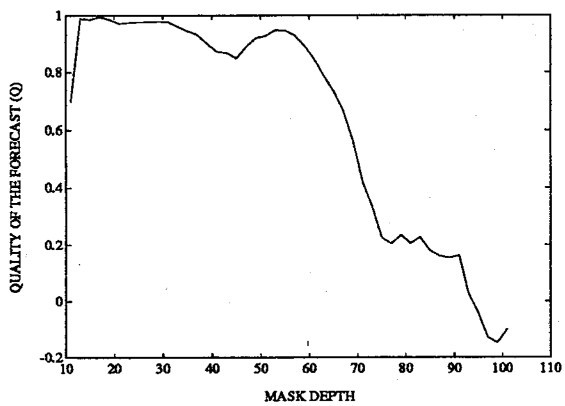

Figure 5: Quality of the Forecast vs. Mask Depth

BIOMEDICAL SYSTEM

It is much more difficult to obtain even half–way decent predictions for biomedical than for technical applications. This is due to the qualitative shape of the correlation functions in the two cases. Whereas the technical systems usually offer wide correlation functions, biomedical correlation functions are often quite narrow.

A biomedical system for predicting the right value of an anaesthetic agent to be applied to patients during surgery is used. The clinical variables comprising heart rate (HR), respiration rate (RR), and systolic arterial pressure (SAP), were selected as the key clinical indicator signals to be used for suggesting an anaesthetic dose (the control signal).

According to information obtained from anaesthetists, the slowest time constant of interest in this system is on the order of 10 minutes, and the fastest time constant of importance is on the order of 1 minute (Nebot et al. 1993).

As in the case of the linear system, the variation in forecast quality as a function of the time span covered by the mask, Δt, will be shown.

$\delta t < t_s, t_l < \Delta t < C_h$

In accordance with previously made recommendations, values of $\delta t = 0.5$ seconds and $\Delta t = 10$ seconds were chosen. Consequently, the mask depth is 21. With this choice, the forecast exhibits indeed the best results that can be obtained for this system. These results are shown in figure 6.

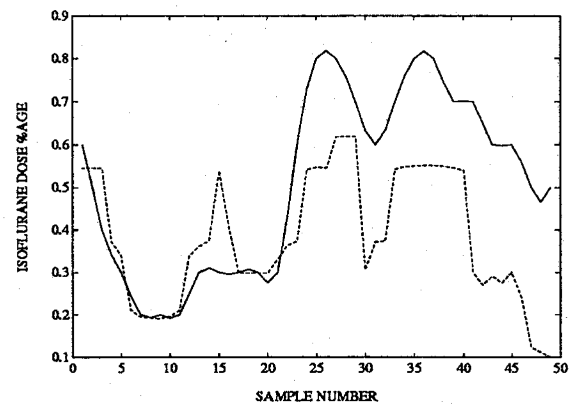

Figure 6: Real vs. Pted. Behavior (Mask Depth 21)

It turns out that, in this example, the causality horizon is just about as large as the largest time constant to be modeled. Thus, the forecast is never as good as in the case of the linear system since the effects of causality degradation set in before the slowest time constants are truly and fully covered by the mask. The slowest time constant is just beyond the causality horizon. The maxima and minima are predicted correctly, but the forecast is not bias free. There is a tendency for drifting away. Figure 7 shows the correlation functions.

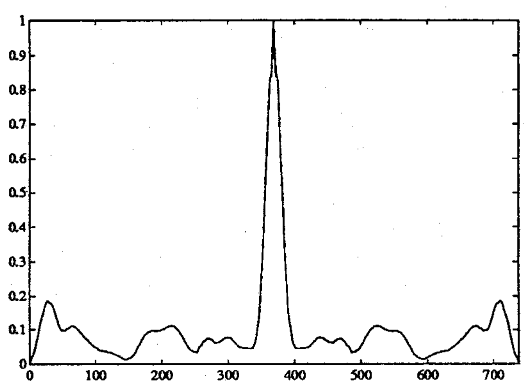

Figure 7: Autocorrelation Function (DOSE)

$\delta t < t_s, t_l, C_h < \Delta t$

When Δt is increased, the quality of the forecasts deteriorates rapidly, and the forecasts look like noise. Figure 8 shows the results obtained with a mask depth of 35. When the mask depth is increased from 25 to 51, $\varepsilon_{\mathrm{RMS}}$ grows proportionally.

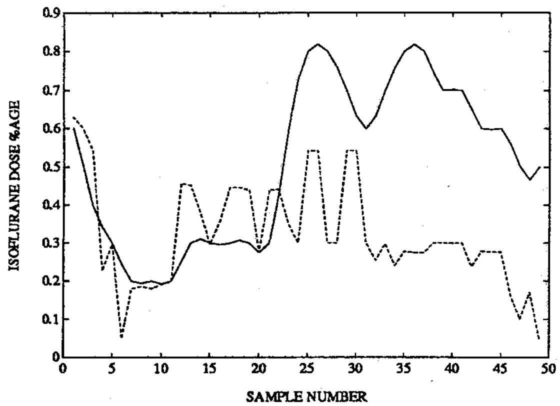

Figure 8: Real vs. Pted. Behavior (Mask Depth 35)

$\delta t, \Delta t < t_s, t_l, C_h$

Here, a sampling rate of $\delta t = 0.2$ minutes was used. For Δt to be smaller than t_s, a mask depth of $depth = 2$ was chosen.

The fuzzy inductive reasoner predicts that the amount of anaesthetic agent to be administered to the patient should be the same that was administered to this patient one sampling period earlier. Therefore, it forecasts a straight line. This result is illustrated in figure 9.

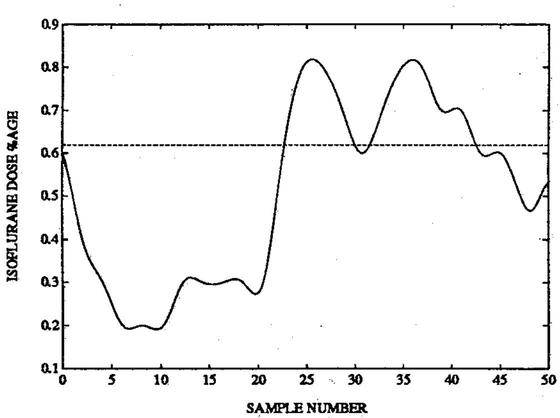

Figure 9: Real vs. Pted. Behavior (Mask Depth 2)

CONCLUSIONS

In this paper, the concept of a causality horizon has been introduced. It has been demostrated by means of two examples that the causality horizon is an important factor influencing the forecast quality. It was shown that the correlation functions are a good indicator for the causality horizon.

The first example is a linear system. In this situation, the causality horizon is quite large, guaranteeing a good prediction if Δt covers both time constants and is not chosen unreasonably large. Several tests were conducted varying Δt, which showed the influence on forecast quality. The forecast quality vs. mask depth was computed and presented in figure 5.

For the second example, a soft science system has been chosen. Such systems don't lend themselves as readily to obtaining good predictions as technical applications do. This is primarily due to the inherent reduction of their causality horizons. This paper illustrates this reduction and explains how it influences the forecast quality.

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