State-Space Realization of a PM Representation

Algorithm:

Given:
$$\left| \begin{aligned} P(s)\cdot \underline{z}(t) &= Q(s)\cdot \underline{u}(t) \\ \underline{y}(t) &= R(s)\cdot \underline{z}(t) + W(s)\cdot \underline{u} \end{aligned} \right.$$

(i) Get $P(s)$ into row-proper form:

$$\underbrace{U_L(s)\cdot P(s)}_{P'(s)}\cdot \underline{z}(t) = \underbrace{U_L(s)\cdot Q(s)}_{Q'(s)}\cdot \underline{u}(t)$$

$P'(s)$ ← row-proper
i.e. $|\Gamma_r[P'(s)]| \neq \emptyset$

$$\Rightarrow \left| \begin{aligned} P'(s)\cdot \underline{z}(t) &= Q'(s)\cdot \underline{u}(t) \\ \underline{y}(t) &= R(s)\cdot \underline{z}(t) + W(s)\cdot \underline{u}(t) \end{aligned} \right|$$

(ii) Use the similarity transformation

$$\underline{z}_0(t) = \Gamma_r\cdot \underline{z}(t) \Rightarrow \underline{z}(t) = \Gamma_r^{-1}\cdot \underline{z}_0(t)$$

$$\Rightarrow \underbrace{P'(s)\cdot \Gamma_r^{-1}}_{P_0(s)}\cdot \underline{z}_0(t) = Q'(s)\cdot \underline{u}(t)$$

$$\underline{y}(t) = \underbrace{R(s) \cdot \Gamma_r^{-1}}_{R_0(s)} \cdot \underline{z}_0(t) + W(s) \cdot \underline{u}(t)$$

$$\Rightarrow \left| \begin{array}{l} P_0(s) \cdot \underline{z}_0(t) = Q'(s) \cdot \underline{u}(t) \\ \underline{y}(t) = R_0(s) \cdot \underline{z}_0(t) + W(s) \cdot \underline{u} \end{array} \right.$$

Notice: $P_0(s)$ is in a special form now:

$$P_0(s) = \begin{bmatrix} (s^{\bar{d}_1} + \cdots) & (\cdots) & \cdots & (\cdots) \\ (\cdots) & (s^{\bar{d}_2} + \cdots) & \cdots & (\cdots) \\ \vdots & \vdots & \ddots & \vdots \\ (\cdots) & (\cdots) & \cdots & (s^{\bar{d}_q} + \cdots) \end{bmatrix}$$

$$\Leftrightarrow \Gamma_r [P_0(s)] = I^{(q)}$$

(If a $\bar{d}_k = \phi \Rightarrow$ We got an unimportant equation that can be eliminated.)

(iii) $\underline{z}_0(s) = P_0^{-1}(s) \cdot Q'(s) \cdot \underline{u}(s) = G_0(s) \cdot$

(iv) If $G_0(s)$ is not strictly proper, divide through $\Rightarrow$

$$\Rightarrow G_0(s) = \underbrace{\overline{G}_0(s)}_{\text{strictly proper}} + \underbrace{H_0(s)}_{\text{polynomial matrix}}$$

(v) Use the transformation:

$$\overline{\underline{z}}_0(t) = \underline{z}_0(t) - H_0(s)\cdot\underline{u}(t)$$

$$\Rightarrow \underline{z}_0(t) = \overline{\underline{z}}_0(t) + H_0(s)\cdot\underline{u}(t)$$

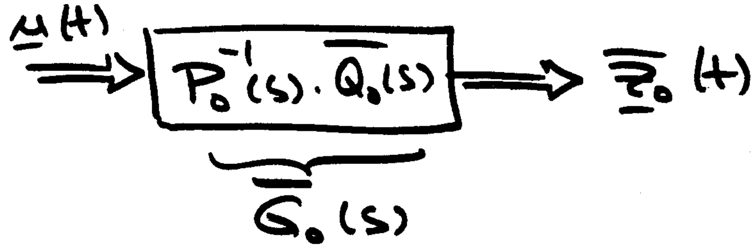
$$\rightarrow P_0(s)\cdot\overline{\underline{z}}_0(t) + P_0(s)\cdot H_0(s)\cdot\underline{u}(t) = Q'(s)\cdot\underline{u}$$

$$\rightarrow P_0(s)\cdot\overline{\underline{z}}_0(t) = (\underbrace{Q'(s) - P_0(s)H_0(s)}_{\overline{Q}_0(s)})\cdot\underline{u}$$

$$\begin{aligned}\underline{y}(t) &= R_0(s)\cdot\overline{\underline{z}}_0(t) + R_0(s)H_0(s)\,\underline{u}(t) + W(s)\\ &= R_0(s)\cdot\overline{\underline{z}}_0(t) + (\underbrace{W(s) + R_0(s)H_0(s)}_{\overline{W}_0(s)})\,\underline{u}\end{aligned}$$

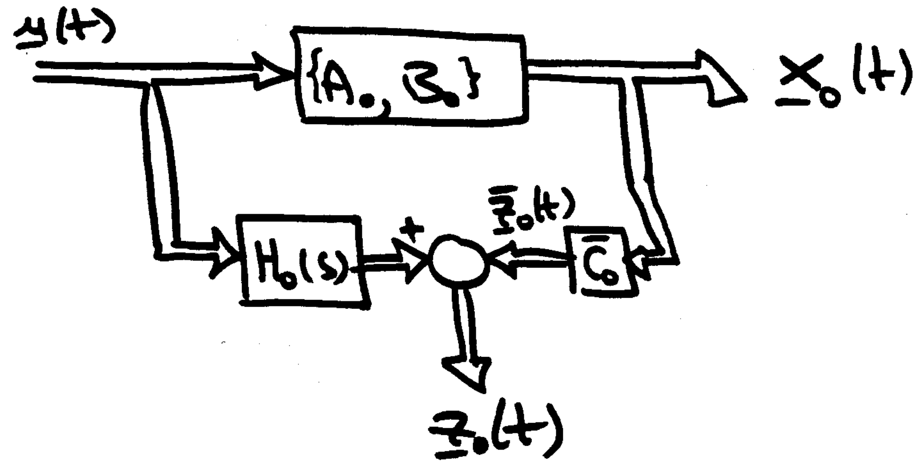
$$\Rightarrow \left|\begin{aligned} P_0(s)\cdot\overline{\underline{z}}_0(t) &= \overline{Q}_0(s)\,\underline{u}(t)\\ \underline{y}(t) &= R_0(s)\,\overline{\underline{z}}_0(t) + \overline{W}_0(s)\cdot\underline{u}(t)\end{aligned}\right.$$

(vi) We wish to realize:



Properties:
- $\overline{G_o}(s)$ is strictly proper
- $P_o(s)$ is row-proper
- $\Gamma_r[P_o(s)] = I^{(q)}$

Find:



⇒ Known problem. Use the observable form of the structure theorem.

Thus: $\overline{G}_o(s) = \left[\overline{\delta}(s)\cdot \overline{C}_q^{-1}\right]^{-1}\cdot\left[\overline{\Sigma}(s)\cdot\overline{B}\right]$

where: $\overline{\delta}(s) = \begin{bmatrix} s^{\bar{d}_1} & & \emptyset \\ & s^{\bar{d}_2} & \\ \emptyset & & \ddots \\ & & & s^{\bar{d}_q}\end{bmatrix} - \overline{\Sigma}(s)\cdot\overline{A}_q$

$$\overline{\Sigma}(s) = \begin{bmatrix} 1 \;\; s \cdots s^{\bar{d}_1 - 1} & & \emptyset \\ & 1 \;\; s \cdots s^{\bar{d}_2 - 1} & \\ \emptyset & & \ddots \\ & & & 1 \;\; s \cdots s^{\bar{d}_q - 1}\end{bmatrix}$$

and: $\overline{C}_q = I^{(q)}$

$P_o(s) = \overline{\delta}(s)$

$\Rightarrow \overline{A}_q$ contains the polynomial coefficients of $P_o(s)$

$\overline{Q}_o(s) = \overline{\Sigma}(s)\cdot\overline{B}$

$\Rightarrow \overline{B}$ contains the polynomial coefficients of $\overline{Q}_o(s)$

$\Longrightarrow \{A_o, B_o, \overline{C}_o\}$ using the previously discussed algorithm.

(vii) We need to undo the transformations:

$$\underline{z}(t) = \Gamma_r^{-1} \cdot \underline{z}_o(t) = \Gamma_r^{-1} \left\{ \overline{\underline{z}}_o(t) + H_o(s) \cdot \underline{u}(t \right.$$

$$\equiv \underbrace{\Gamma_r^{-1} \cdot \overline{C}_o}_{C_o} \cdot \underline{x}_o(t) + \underbrace{\Gamma_r^{-1} \cdot H_o(s)}_{H(s)} \cdot \underline{u}(t)$$

$$\Rightarrow \left| \begin{array}{l} \dot{\underline{x}}_o(t) = A_o \underline{x}_o(t) + B_o \underline{u}(t) \\ \underline{z}(t) = C_o \underline{x}_o(t) + H(s) \cdot \underline{u}(t) \end{array} \right|$$

(viii) $\underline{y}(t) = R(s) \cdot \underline{z}(t) + W(s) \cdot \underline{u}(t)$

$$= \underbrace{R(s)\, C_o \cdot \underline{x}_o(t)}_{\substack{\text{may contain} \\ \text{derivatives of } \underline{x}_o(t)}} + \big(W(s) + R(s) \cdot H(s)\big)$$

$\Rightarrow$ eliminate from:

$$\dot{\underline{x}}_o(t) = A_o \underline{x}_o(t) + B_o \underline{u}(t)$$

$$\Longrightarrow \left| \begin{array}{l} \dot{\underline{x}}_o(t) = A_o \underline{x}_o(t) + B_o \underline{u}(t) \\ \underline{y}(t) = C\, \underline{x}_o(t) + D(s)\, \underline{u}(t) \end{array} \right|$$

Notice: Due to all the transformat[ions] there is no guarantee tha[t] this representation is still fully observable.

Notice: This algorithm proves a lemma which, until now, we simply assumed to be correct:

Lemma: Every PM-Representati[on] has an equivalent State-Space realization (possibly with $D(s)$)

$\Rightarrow$ Examples of this algorithm are in Wolovich pp. 146-151

Example:

$$\left| \begin{aligned} P(s)\cdot \underline{z}(t) &= Q(s)\cdot \underline{u}(t) \\ \underline{y}(t) &= R(s)\cdot \underline{z}(t) + W(s)\cdot \underline{u}(t) \end{aligned} \right|$$

where:

$$P(s) = \begin{bmatrix} (s^3-1) & (-s^3+1) & \phi \\ (s^2+s) & (-s^2+1) & (s-1) \\ (s^2+s) & (-s^2-1) & (s+1) \end{bmatrix}$$

$$Q(s) = \begin{bmatrix} (s^3-2s) & (s^2+3s) \\ (\tfrac{1}{2}s-\tfrac{3}{2}) & (s+2) \\ (-\tfrac{1}{2}s-\tfrac{5}{2}) & (s+4) \end{bmatrix}$$

$$R(s) = \begin{bmatrix} (s-1) & (s+2) & (-2s-3) \\ s & -s & \phi \\ (s+1) & (-s+1) & 2 \end{bmatrix}$$

$$W(s) = \phi^{(3\times 2)}$$

(i) Make $P(s)$ row-proper:

$$\Gamma_r[P(s)] = \begin{bmatrix} 1 & -1 & \phi \\ 1 & -1 & \phi \\ 1 & -1 & \phi \end{bmatrix}$$

$\Rightarrow \text{Rank}\{\Gamma_r[P(s)]\} = 1 \implies$ not row-prop

We find any unimodular matrix that makes $P'(s)$ row-proper, e.g.:

$$U_L(s) = \begin{bmatrix} 1 & -s & \emptyset \\ \emptyset & 1 & -1 \\ \emptyset & \emptyset & 1 \end{bmatrix}$$

(Several algorithms for this problem were already presented).

$$\Rightarrow P'(s) = U_L(s) \cdot P(s) = \begin{bmatrix} (-s^2-1) & (-s+1) & (-s^2+\cdots \\ \emptyset & 2 & -2 \\ (s^2+s) & (-s^2-1) & (s+\cdots \end{bmatrix}$$

$$Q'(s) = U_L(s) \cdot Q(s) = \begin{bmatrix} (s^3 - \frac{1}{2}s^2 - \frac{1}{2}s) & s \\ (s+1) & -2 \\ (-\frac{1}{2}s - \frac{5}{2}) & (s+4) \end{bmatrix}$$

$$\left| \; P'(s) \cdot \underline{z}(t) = Q'(s) \cdot \underline{u}(t) \; \right|$$

is the same problem as before, but:

$$\Gamma_r\left[P'(s)\right] = \begin{bmatrix} -1 & \emptyset & -1 \\ \emptyset & 2 & -2 \\ 1 & -1 & \emptyset \end{bmatrix}$$

$$\det\left\{\Gamma_r\left[P'(s)\right]\right\} = 4 \neq \emptyset$$

$$\Rightarrow P'(s) \text{ is row-proper.}$$

$$\Rightarrow \Gamma_r^{-1} = \frac{\begin{bmatrix} -2 & 1 & 2 \\ -2 & 1 & -2 \\ -2 & -1 & -2 \end{bmatrix}}{4}$$

(ii) $\underline{z}_0(t) = \Gamma_r \cdot \underline{z}(t)$

$$\Rightarrow P_0(s) = P'(s) \cdot \Gamma_r^{-1} = \begin{bmatrix} s^2 & -\frac{1}{2}s & -1 \\ \phi & 1 & \phi \\ -s & -\frac{1}{2} & s^2 \end{bmatrix}$$

$$R_0(s) = R(s) \cdot \Gamma_r^{-1} = \begin{bmatrix} 1 & (s+1) & s \\ \phi & \phi & s \\ -2 & \phi & (s-1) \end{bmatrix}$$

We find that $\overline{d}_2 = \phi$

$\Rightarrow$ we have an unimportant equation.

$\Rightarrow$ Apply row-operations to nullify all off-diagonal elements of the unimportant column:

$$U_{L_2}(s) = \begin{bmatrix} 1 & \frac{1}{2}s & \phi \\ \phi & 1 & \phi \\ \phi & \frac{1}{2} & 1 \end{bmatrix}$$

$$\Rightarrow P_0'(s) = U_{L_2}(s)\cdot P_0(s) = \begin{bmatrix} s^2 & \phi & -1 \\ \phi & 1 & \phi \\ -s & \phi & s^2 \end{bmatrix}$$

$$Q_0'(s) = U_{L_2}(s)\cdot Q'(s) = \begin{bmatrix} s^3 & \phi \\ (s+1) & -2 \\ -2 & (s+3) \end{bmatrix}$$

$$\Rightarrow z_{0_2} = (s+1)u_1 - 2u_2 = u_1 + \dot{u}_1 - 2u_2$$

$$y_1 = z_{01} + (s+1)z_{02} + s\cdot z_{03}$$
$$= z_{01} + z_{02} + \dot{z}_{02} + \dot{z}_{03}$$
$$= z_{01} + u_1 + \dot{u}_1 - 2u_2 + \dot{u}_1 + \ddot{u}_1 - 2\dot{u}_2 + \dot{}$$
$$= z_{01} + s\cdot z_{03} + (1+2s+s^2)u_1 + (-2-2s)$$

$$\Rightarrow y_1 = \begin{bmatrix} 1 & s \end{bmatrix}\cdot\begin{bmatrix} z_{01} \\ z_{03} \end{bmatrix} + \begin{bmatrix} (1+2s+s^2) & (-2-2s) \end{bmatrix}\begin{bmatrix} u \\ u \end{bmatrix}$$

$$\Rightarrow \left|\begin{aligned} P_0''(s)\cdot \underline{z}_{0_r}(t) &= Q_0''(s)\cdot \underline{u}(t) \\ y(t) &= R_0''(s)\cdot \underline{z}_{0_r}(t) + W_0''(s)\cdot \underline{u}(t) \end{aligned}\right.$$

where: $\underline{z}_{0_r}(t) = \begin{bmatrix} z_{01} \\ z_{03} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}\cdot \underline{z}_0(t)$

$$P_o''(s) = \begin{bmatrix} s^2 & -1 \\ -s & s^2 \end{bmatrix} ; \quad Q_o''(s) = \begin{bmatrix} s^3 & \emptyset \\ -2 & (s+3) \end{bmatrix}$$

$$R_o''(s) = \begin{bmatrix} 1 & s \\ \emptyset & s \\ -2 & (s-1) \end{bmatrix} ; \quad W_o''(s) = \begin{bmatrix} (s^2+2s+1) & (-2s \\ \emptyset & \emptyset \\ \emptyset & \emptyset \end{bmatrix}$$

Notice that none of these transformations affected the properties of $P_o''(s)$:

$$\Gamma_r\left[P_o''(s)\right] = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \end{bmatrix} = I^{(2)}$$

(iii) $G_o''(s) = P_o''(s)^{-1} \cdot Q_o''(s)$

$$= \frac{\begin{bmatrix} s^2 & 1 \\ s & s^2 \end{bmatrix} \cdot \begin{bmatrix} s^3 & \emptyset \\ -2 & (s+3) \end{bmatrix}}{s^4 - s} = \frac{\begin{bmatrix} (s^5-2) & (s+3 \\ (s^4-2s^2) & (s^3+3s \end{bmatrix}}{s^4-s}$$

(iv)

$$\Rightarrow G_o''(s) = \overline{G}_o(s) + H_o(s)$$

$$H_o(s) = \begin{bmatrix} s & \emptyset \\ 1 & \emptyset \end{bmatrix} ; \quad \overline{G}_o(s) = \frac{\begin{bmatrix} (+s^2-2) & (s+3 \\ (-2s^2+s) & (s^3+3s \end{bmatrix}}{s^4-s}$$

(v) $\underline{\bar{z}}_0(t) = \underline{z}_{o_r}(t) - H_o(s)\cdot \underline{y}(t)$

$$\Rightarrow \left| \begin{aligned} P_o''(s)\cdot \underline{\bar{z}}_0(t) &= \bar{Q}_o(s)\cdot \underline{u}(t) \\ \underline{y}(t) &= R_o''(s)\cdot \underline{\bar{z}}_0(t) + \bar{W}_o(s)\cdot \underline{u}(t) \end{aligned} \right.$$

where:

$$\bar{Q}_o(s) = Q_o''(s) - P_o''(s)\,.\,H_o(s)$$

$$= \begin{bmatrix} s^3 & \phi \\ -2 & (s+3) \end{bmatrix} - \begin{bmatrix} s^2 & -1 \\ -s & s^2 \end{bmatrix} \cdot \begin{bmatrix} s & \phi \\ 1 & \phi \end{bmatrix}$$

$$= \begin{bmatrix} s^3 & \phi \\ -2 & (s+3) \end{bmatrix} - \begin{bmatrix} (s^3-1) & \phi \\ \phi & \phi \end{bmatrix} = \begin{bmatrix} 1 & \phi \\ -2 & (s+3) \end{bmatrix}$$

$$\bar{W}_o(s) = W_o''(s) + R_o''(s)\cdot H(s)$$

$$= \begin{bmatrix} (s^2+2s+1) & (-2s-2) \\ \phi & \phi \\ \phi & \phi \end{bmatrix} + \begin{bmatrix} 1 & s \\ \phi & s \\ -2 & (s-1) \end{bmatrix} \cdot \begin{bmatrix} s & \phi \\ 1 & \phi \end{bmatrix}$$

$$= \begin{bmatrix} (s^2+2s+1) & (-2s-2) \\ \phi & \phi \\ \phi & \phi \end{bmatrix} + \begin{bmatrix} 2s & \phi \\ s & \phi \\ (-s-1) & \phi \end{bmatrix}$$

$$= \begin{bmatrix} (s^2+4s+1) & (-2s-2) \\ s & \phi \\ (-s-1) & \phi \end{bmatrix}$$

$$\overline{G}_o(s) = P_o''^{-1}(s) \cdot \overline{Q}_o(s)$$

$$= \frac{\begin{bmatrix} s^2 & 1 \\ s & s^2 \end{bmatrix} \cdot \begin{bmatrix} 1 & \emptyset \\ -2 & (s+3) \end{bmatrix}}{(s^4 - s)} = \frac{\begin{bmatrix} (s^2-2) & (s+3) \\ (-2s^2+s) & (s^3+3s^2 \end{bmatrix}}{(s^4-s)}$$

as expected.

$$P_o''(s) = \begin{bmatrix} s^2 & -1 \\ -s & s^2 \end{bmatrix} = \begin{bmatrix} s^2 & \emptyset \\ \emptyset & s^2 \end{bmatrix} - \begin{bmatrix} \emptyset & 1 \\ s & \emptyset \end{bmatrix}$$

$$= \begin{bmatrix} s^2 & \emptyset \\ \emptyset & s^2 \end{bmatrix} - \begin{bmatrix} 1 & s & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & s \end{bmatrix} \begin{bmatrix} \emptyset & 1 \\ \emptyset & \emptyset \\ \emptyset & \emptyset \\ 1 & \emptyset \end{bmatrix}$$

$$= \begin{bmatrix} s^2 & \emptyset \\ \emptyset & s^2 \end{bmatrix} = \overline{\Sigma}(s) \cdot \overline{A}_q$$

$$\Rightarrow A_o = \begin{bmatrix} \emptyset & \emptyset & \emptyset & 1 \\ 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & 1 & \emptyset \end{bmatrix}$$

$$\overline{C}_q = I^{(2)}$$

$$\Rightarrow \overline{C}_o = \begin{bmatrix} \emptyset & 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix}$$

$$\overline{Q}_o(s) = \overline{\overline{\Sigma}}(s) \cdot \overline{B}$$

$$= \begin{bmatrix} 1 & \phi \\ -2 & (s+3) \end{bmatrix} = \begin{bmatrix} 1 & s & \phi & \phi \\ \phi & \phi & 1 & s \end{bmatrix} \cdot \begin{bmatrix} 1 & \phi \\ \phi & \phi \\ -2 & 3 \\ \phi & 1 \end{bmatrix}$$

$$\overline{B} = B_o = \begin{bmatrix} 1 & \phi \\ \phi & \phi \\ -2 & 3 \\ \phi & 1 \end{bmatrix}$$

$$\Rightarrow \left| \begin{array}{l} \underline{\dot{x}} = \begin{bmatrix} \phi & \phi & \phi & 1 \\ 1 & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi \\ \phi & 1 & 1 & \phi \end{bmatrix} \underline{x} + \begin{bmatrix} 1 & \phi \\ \phi & \phi \\ -2 & 3 \\ \phi & 1 \end{bmatrix} \underline{u} \\ \overline{\underline{z}}_o = \begin{bmatrix} \phi & 1 & \phi & \phi \\ \phi & \phi & \phi & 1 \end{bmatrix} \cdot \underline{x} \end{array} \right|$$

is an observer-canonical realizatio

(vii) We need to undo all the transformations:

(a) $\underline{z}_{o_r} = \overline{\underline{z}}_o + H_o(s) \cdot \underline{u}(t)$

$$\Rightarrow \underline{z}_{0r} = \begin{bmatrix} \phi & 1 & \phi & \phi \\ \phi & \phi & \phi & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} s & \phi \\ 1 & \phi \end{bmatrix} \underline{u}$$

(b) $\underline{z}_0 = \begin{bmatrix} z_{01} \\ z_{02} \\ z_{03} \end{bmatrix}$; $\underline{z}_{0r} = \begin{bmatrix} z_{01} \\ z_{03} \end{bmatrix}$

__where__ : $z_{02} = \begin{bmatrix} (s+1) & -2 \end{bmatrix} \underline{u}$

$$\Rightarrow \underline{z}_0 = \begin{bmatrix} \phi & 1 & \phi & \phi \\ \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} s & \phi \\ (s+1) & -2 \\ 1 & \phi \end{bmatrix} \underline{u}$$

(c) $\underline{z} = \Gamma_r^{-1} \cdot \underline{z}_0 = \dfrac{\begin{bmatrix} -2 & 1 & 2 \\ -2 & 1 & -2 \\ -2 & -1 & -2 \end{bmatrix}}{4} \cdot \underline{z}_0$

$$\Rightarrow \underline{z} = \begin{bmatrix} \phi & -1/2 & \phi & 1/2 \\ \phi & -1/2 & \phi & -1/2 \\ \phi & -1/2 & \phi & -1/2 \end{bmatrix} \underline{x} + \begin{bmatrix} (-1/4\, s + 3/4) & -1/2 \\ (-1/4\, s - 1/4) & -1/2 \\ (-3/4\, s - 3/4) & 1/2 \end{bmatrix} \underline{u}$$

Now, employ the output equation:

$$\underline{y}(t) = R(s) \cdot \underline{z}(t) + W(s) \cdot \underline{u}(t)$$

$$R(s) = \begin{bmatrix} (s-1) & (s+2) & (-2s-3) \\ s & -s & \phi \\ (s+1) & (-s+1) & 2 \end{bmatrix}$$

$$W(s) = \phi^{(3\times 2)}$$

$$\underline{y}(t) = \begin{bmatrix} \phi & 1 & \phi & s \\ \phi & \phi & \phi & s \\ \phi & -2 & \phi & (s-1) \end{bmatrix} \underline{x} + \begin{bmatrix} (s^2+4s+1) & (-2 \\ s & \phi \\ (-s-1) & \phi \end{bmatrix}$$

Get the derivatives out of C:

$$\dot{x}_4 = x_2 + x_3 + u_2 = s x_4$$

$$(s-1)x_4 = x_2 + x_3 - x_4 + u_2$$

$$\Rightarrow \underline{y}(t) = \begin{bmatrix} \phi & 2 & 1 & \phi \\ \phi & 1 & 1 & \phi \\ \phi & -1 & 1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} (s^2+4s+1) & (-2s- \\ s & 1 \\ (-s-1) & 1 \end{bmatrix}$$

$\Rightarrow$ The desired state-space description is

$$\underline{\dot{x}} = \begin{bmatrix} \phi & \phi & \phi & 1 \\ 1 & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi \\ \phi & 1 & 1 & \phi \end{bmatrix} \underline{x} + \begin{bmatrix} 1 & \phi \\ \phi & \phi \\ -2 & 3 \\ \phi & 1 \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} \phi & 2 & 1 & \phi \\ \phi & 1 & 1 & \phi \\ \phi & -1 & 1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} (s^2+4s+1) & (-2s-1) \\ s & 1 \\ (-s-1) & 1 \end{bmatrix} \underline{u}$$

which is no longer in observer-canonical form.

Polynomial - Matrix Realizations of G(

- If $G(s)$ is not strictly proper, divide through:

$$G(s) = \underbrace{\bar{G}(s)}_{\text{strictly proper}} + W(s)$$

(1) Controller - canonical PM-realizati

Find the smallest common multiples (SCM) of each column of $\bar{G}(s)$:

$$\bar{G}(s) = \begin{bmatrix} \frac{r_{11}(s)}{g_1(s)} & \frac{r_{12}(s)}{g_2(s)} & \cdots & \frac{r_{1m}(s)}{g_m(s)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{r_{p1}(s)}{g_1(s)} & \frac{r_{p2}(s)}{g_2(s)} & \cdots & \frac{r_{pm}(s)}{g_m(s)} \end{bmatrix}$$

This can be rewritten as:

$$\bar{G}(s) = \begin{bmatrix} r_{11}(s) & r_{12}(s) & \cdots & r_{1m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ r_{p1}(s) & r_{p2}(s) & \cdots & r_{pm}(s) \end{bmatrix} \cdot \begin{bmatrix} g_1(s) & & & \\ & g_2(s) & & \\ \emptyset & & \ddots & \\ & & & g_{\cdot} \end{bmatrix}$$

This can immediately be interpreted as:

$$G(s) = R(s)\cdot P^{-1}(s)\cdot Q(s) + W(s)$$

where: $R(s) = \begin{bmatrix} r_{11}(s) & r_{12}(s) & \cdots & r_{1m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ r_{p1}(s) & r_{p2}(s) & \cdots & r_{pm}(s) \end{bmatrix}$

$$P(s) = \begin{bmatrix} g_1(s) & & \emptyset \\ & g_2(s) & \\ \emptyset & & \ddots \\ & & & g_m(s) \end{bmatrix}$$

$$Q(s) = I^{(m)}$$

$W(s)$ from division at the beginning.

$\Rightarrow$ This is a controller-canonical PM-realization.

We can then apply the structure theorem to find a controller-canonical time domain representation.

(2) Observer - canonical PM - realization

Find the smallest common multiples (SCM) of each row of $\bar{G}(s)$:

$$\bar{G}(s) = \begin{bmatrix} \frac{q_{11}(s)}{g_1(s)} & \frac{q_{12}(s)}{g_1(s)} & \cdots & \frac{q_{1m}(s)}{g_1(s)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{q_{p1}(s)}{g_p(s)} & \frac{q_{p2}(s)}{g_p(s)} & \cdots & \frac{q_{pm}(s)}{g_p(s)} \end{bmatrix}$$

This can be rewritten as:

$$\bar{G}(s) = \begin{bmatrix} g_1(s) & & & \emptyset \\ & g_2(s) & & \\ \emptyset & & \ddots & \\ & & & g_p(s) \end{bmatrix}^{-1} \cdot \begin{bmatrix} q_{11}(s) & q_{12}(s) & \cdots & q_{1m} \\ q_{21}(s) & q_{22}(s) & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{p1}(s) & q_{p2}(s) & \cdots & q_{pm} \end{bmatrix}$$

$$\Rightarrow G(s) = R(s) \cdot P^{-1}(s) \cdot Q(s) + W(s)$$

<u>where</u>: $R(s) = I^{(p)}$

$$P(s) = \begin{bmatrix} g_1(s) & & & \emptyset \\ & g_2(s) & & \\ \emptyset & & \ddots & \\ & & & g_p(s) \end{bmatrix} ; \quad Q(s) = \begin{bmatrix} q_{11}(s) & \cdots & q_{1m} \\ \vdots & \ddots & \vdots \\ q_{p1}(s) & \cdots & q_{pm} \end{bmatrix}$$

$W(s)$ from division.

$\Longrightarrow$ Observer - canonical realization