```
[> //
[> // Start by defining the system
[> //

[> f = [ -13.4018574943  18.1401915356   1.8764807038  15.4005327260
[>        12.9649863844 -15.0893766561  -2.0844344996 -13.5002136564
[>       -13.5409517386  16.7370866019   3.1734047293  14.1172559212
[>       -27.1560839913  33.9084839810   4.0608462828  29.5847743391 ];

[> g = [    0.1045000346
[>         -0.0993752551
[>          0.1043185134
[>          0.2038710857 ];

[> h = [ -27 51 -18 48 ];

[> i = 0;

[> //
[> // Before we can do anything, we need to check whether this system
[> // is fully controllable and observable
[> //

[> LONG

[> qc = [ g f*g f*f*g f**3*g ];

[> RANK(qc)

ANS          =

    4.

[> SVD(qc)

ANS          =

  0.958435365011706
  0.013170067775898
  0.001086612917261
  0.000008155091290

[> qo = [h ; h*f ; h*f*f ; h*f**3 ];

[> RANK(qo)

ANS          =

    4.

[> SVD(qo)
```

```
ANS             =

    1.0D+02 *

   2.690851023940072
   0.153516732905454
   0.004747938114415
   0.000000000000199

[> RANK(qo,1E-10)

ANS             =

     3.

[> SHORT

[> //
[> // There is one unobservable mode. Perform output decoupling.
[> //

[> qob = [ qo(1:3,:) ; [ 1 0 0 0 ] ];

[> SVD(qob)

ANS             =

  186.1256
    7.6564
    0.8543
    0.0923

[> t = qob

T               =

  -27.0000   51.0000  -18.0000   48.0000
  -36.6904   66.9963  -19.1718   61.6333
  -53.7880   92.5071  -19.0549   83.2369
    1.0000    0.0000    0.0000    0.0000

[> fh = t*f/t

FH              =

    0.0000    1.0000    0.0000    0.0000
    0.0000    0.0000    1.0000    0.0000
    1.6487   -4.3403    3.6604    0.0000
    2.1653   -4.0660    1.9471    0.6065

[> gh = t*g

GH              =
```

```
   0.0184
   0.0733
   0.1680
   0.1045

[> hh = h/t

HH              =

    1.    0.    0.    0.

[> ih = i

IH              =

    0.

[> //
[> // Okay! The last mode is unobservable. Reduce the model.
[> //

[> fr = fh(1:3,1:3)

FR              =

   0.0000    1.0000    0.0000
   0.0000    0.0000    1.0000
   1.6487   -4.3403    3.6604

[> gr = gh(1:3)

GR              =

   0.0184
   0.0733
   0.1680

[> hr = hh(1:3)

HR              =

    1.    0.    0.

[> ir = ih

IR              =

    0.

[> //
[> // Get the "model" into controller-canonical form (why not)
[> //

[> qc = [ gr fr*gr fr*fr*gr ];
```

```
[> qcin = INV(qc);

[> q = qcin(3,:);

[> t = [ q ; q*fr ; q*fr*fr ];

[> fm = t*fr/t

FM              =

   0.0000    1.0000    0.0000
   0.0000    0.0000    1.0000
   1.6487   -4.3403    3.6604

[> gm = t*gr

GM              =

   0.0000
   0.0000
   1.0000

[> hm = hr/t

HM              =

  -0.0204    0.0058    0.0184

[> im = ir

IM              =

    0.

[> //
[> //  Ready for controller/observer design.
[> //  Let's try pole placement first
[> //

[> pc = [ -6 ; -4+4*j ; -4-4*j ]

PC              =

  -6.0000 + 0.0000i
  -4.0000 + 4.0000i
  -4.0000 - 4.0000i

[> pd = EXP(pc*0.1)

PD              =

   0.5488 + 0.0000i
   0.6174 + 0.2610i
   0.6174 - 0.2610i
```

```
[> k1 = PLACE(fm,gm,pd)

K1          =

   1.4021   -3.2133    1.8768

[> fcl = fm - gm*k1;

[> EIG(fcl)

ANS         =

   0.5488 - 0.0000i
   0.6174 + 0.2610i
   0.6174 - 0.2610i

[> poc = [-8 ; -6+6*j ; -6-6*j ];

[> pod = EXP(poc*0.1)

POD         =

   0.4493 + 0.0000i
   0.4530 + 0.3099i
   0.4530 - 0.3099i

[> l1 = PLACE(fm',hm',pod)'

L1          =

-146.3522
 -96.6574
  -6.1536

[> //
[> //  L1 is too big.  Let's try to balance.
[> //

[> t = SQRT(ABS(k1'./l1))

T           =

   0.0979
   0.1823
   0.5523

[> t = DIAG(t)

T           =

   0.0979    0.0000    0.0000
   0.0000    0.1823    0.0000
   0.0000    0.0000    0.5523
```

```
[> fn = t*fm/t

FN              =

    0.0000     0.5368     0.0000
    0.0000     0.0000     0.3302
    9.3024   -13.1464     3.6604

[> gn = t*gm

GN              =

    0.0000
    0.0000
    0.5523

[> hn = hm/t

HN              =

   -0.2083     0.0321     0.0334

[> in = im

IN              =

     0.

[> k1 = PLACE(fn,gn,pd)

K1              =

   14.3249   -17.6236     3.3984

[> l1 = PLACE(fn',hn',pod)'

L1              =

 -14.3249
 -17.6236
  -3.3984

[> //
[> //  Fine.  That worked.  Let's try DLQR instead.
[> //

[> DEFF dlqry2

[> q = 1

Q               =

     1.
```

```
[> r = 1

R               =

     1.

[> sc = 4

SC              =

     4.

[> sd = EXP(sc*0.1)

SD              =

    1.4918

[> ft = sd*fm; gt = sd*gm;

[> k2 = DLQRY2(ft,gt,hm,im,q,r)

K2              =

    1.5938    -3.8925     2.4782

[> fcl2 = fm - gm*k2;

[> EIG(fcl2)

ANS             =

    0.3029 + 0.0000i
    0.3308 - 0.0000i
    0.5485 + 0.0000i

[> so = 6;

[> sod = EXP(so*0.1)

SOD             =

    1.8221

[> ft = sod*fm; gt = sod*gm;

[> l2 = DLQRY2(ft',hm',gt',im',q,r)'

L2              =

-521.1048
-395.3927
-167.4049
```

```
[> //
[> //  L2 is also too large.  Needs also balancing
[> //

[> t = SQRT(ABS(k2'./l2))

T           =

   0.0553
   0.0992
   0.1217

[> t = DIAG(t)

T           =

   0.0553    0.0000    0.0000
   0.0000    0.0992    0.0000
   0.0000    0.0000    0.1217

[> fn2 = t*fm/t

FN2         =

   0.0000    0.5574    0.0000
   0.0000    0.0000    0.8155
   3.6273   -5.3224    3.6604

[> gn2 = t*gm

GN2         =

   0.0000
   0.0000
   0.1217

[> hn2 = hm/t

HN2         =

  -0.3686    0.0589    0.1516

[> in2 = im

IN2         =

    0.

[> ft = sd*fn2; gt = sd*gn2;

[> k2 = DLQRY2(ft,gt,hn2,in2,q,r)

K2          =
```

```
  28.8187  -39.2312   20.3683

[> ft = sod*fn2; gt = sod*gn2;

[> l2 = DLQRY2(ft',hn2',gt',in2',q,r)'

L2          =

 -28.8187
 -39.2312
 -20.3683

[> //
[> //  Let us check the parameter sensitivity of the two designs.
[> //

[> k1

K1          =

  14.3249  -17.6236    3.3984

[> k1m = k1 + 0.01*NORM(k1)*RAND(k1)

K1M         =

  14.3735  -17.4500    3.3984

[> fcl = fn - gn*k1;

[> fclm = fn - gn*k1m;

[> EIG(fcl)

ANS         =

   0.5488 - 0.0000i
   0.6174 + 0.2610i
   0.6174 - 0.2610i

[> EIG(fclm)

ANS         =

   0.3989 - 0.0000i
   0.6924 + 0.3563i
   0.6924 - 0.3563i

[> ABS(EIG(fcl))

ANS         =

   0.5488
   0.6703
```

```
   0.6703

[>  ABS(EIG(fclm))

ANS          =

   0.3989
   0.7787
   0.7787

[>  //

[>  k2

K2           =

  28.8187  -39.2312   20.3683

[>  k2m = k2 + 0.01*NORM(k2)*RAND(k2)

K2M          =

  28.9930  -38.8801   20.6999

[>  fcl2 = fn2 - gn2*k2;

[>  fcl2m = fn2 - gn2*k2m;

[>  EIG(fcl2)

ANS          =

   0.3029 + 0.0000i
   0.3308 - 0.0000i
   0.5485 + 0.0000i

[>  EIG(fcl2m)

ANS          =

   0.1287 + 0.0000i
   0.5066 + 0.3092i
   0.5066 - 0.3092i

[>  ABS(EIG(fcl2))

ANS          =

   0.3029
   0.3308
   0.5485

[>  ABS(EIG(fcl2m))
```

```
ANS          =

    0.1287
    0.5935
    0.5935

[>  //

[>  DIARY -off
```