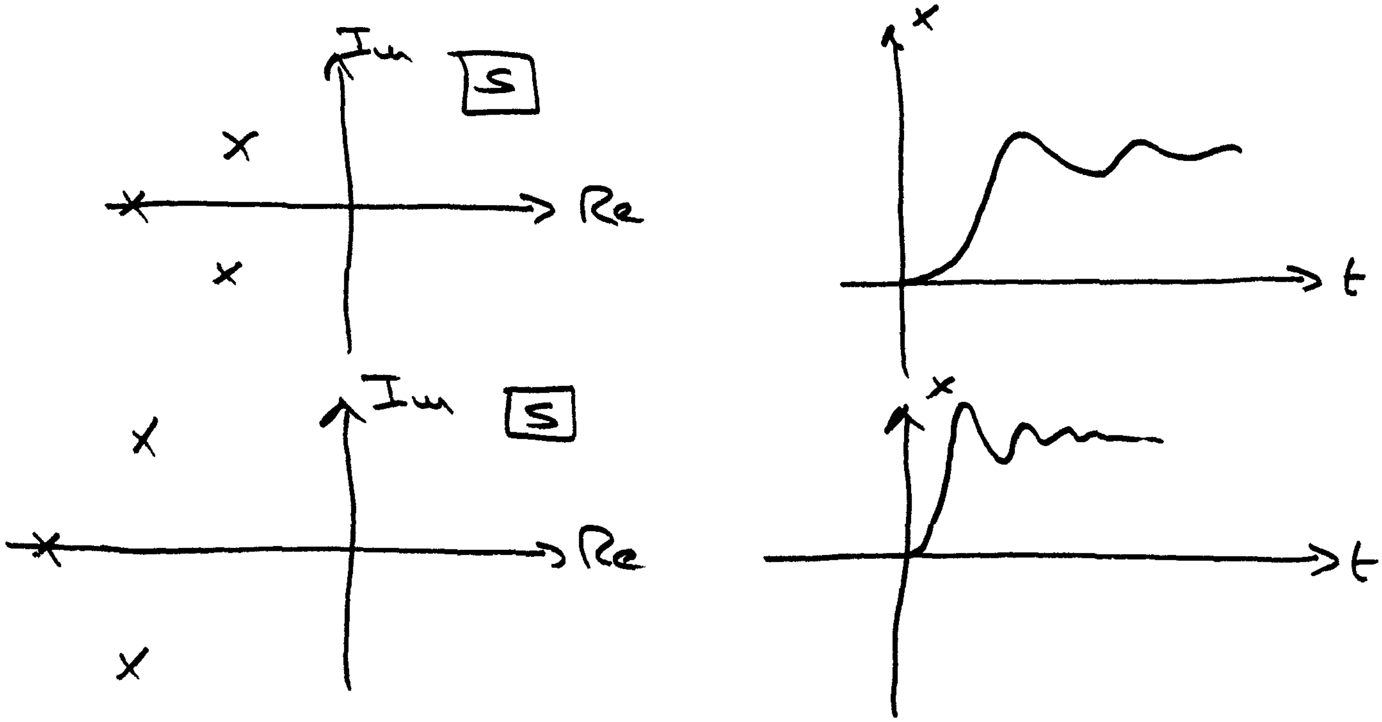
## Deadbeat Control:

We can make a continuous system faster and faster by moving the poles further and further to the left:




In the limit, all poles will be infinitely far to the left. In this case, the system response is immediate.

The problem with this design is the cost of its implementation. If you design a full state feedback, the values of the $\underline{k}'$-vector will grow beyond all bounds.

$\Rightarrow$ It takes an infinite amount of gas to accelerate a car infinitely fast.

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A reasonable question that deserves to be discussed is what happens if we sample such a system, i.e., if we move the design from the s-plane to the z-plane.

A system with all closed-loop continuous poles at infinity corresponds to a system with all closed-loop discrete poles at the origin:

$$\lambda_c \to -\infty \iff \lambda_d = e^{\lambda_c T} = \emptyset .$$

Thus:

$$\boxed{G_{tot}(z) = \frac{P_{tot}(z)}{z^n}}$$

Example:

$$G_{tot}(z) = \frac{\frac{8}{3} z^3 - \frac{4}{3} z^2 - \frac{2}{3} z + \frac{1}{3}}{z^4}$$

All the poles (and even all of the zeroes) are inside the unit circle, thus this system is stable.

DC-gain:

$$\lim_{z \to 1} G_{tot}(z) = 1.\emptyset$$

$\Rightarrow$ If I apply a unit step to this system, I expect the response to approach a value of 1.

$$G_{tot}(z) = \frac{Y(z)}{U(z)} \quad ; \quad U(z) = \frac{z}{z-1}$$

$$\Rightarrow Y(z) = G_{tot}(z) \cdot U(z) = \frac{\frac{8}{3}z^3 - \frac{4}{3}z^2 - \frac{2}{3}z + \frac{1}{3}}{z^3(z-1)}$$

$$\Rightarrow Y(z^{-1}) = \frac{\frac{8}{3}z^{-1} - \frac{4}{3}z^{-2} - \frac{2}{3}z^{-3} + \frac{1}{3}z^{-4}}{1 - z^{-1}}$$

Let us use the division algorithm:

$$\left(\frac{8}{3}z^{-1}-\frac{4}{3}z^{-2}-\frac{2}{3}z^{-3}+\frac{1}{3}z^{-4}\right):\left(1-z^{-1}\right)=\frac{8}{3}z^{-1}+\frac{4}{3}z^{-2}+\frac{2}{3}z^{-3}+z^{-4}+z^{-5}+\dots$$

$$-\quad\frac{8}{3}z^{-1}-\frac{8}{3}z^{-2}$$

$$\frac{4}{3}z^{-2}-\frac{2}{3}z^{-3}$$

$$-\quad\frac{4}{3}z^{-2}-\frac{4}{3}z^{-3}$$

$$\frac{2}{3}z^{-3}+\frac{1}{3}z^{-4}$$

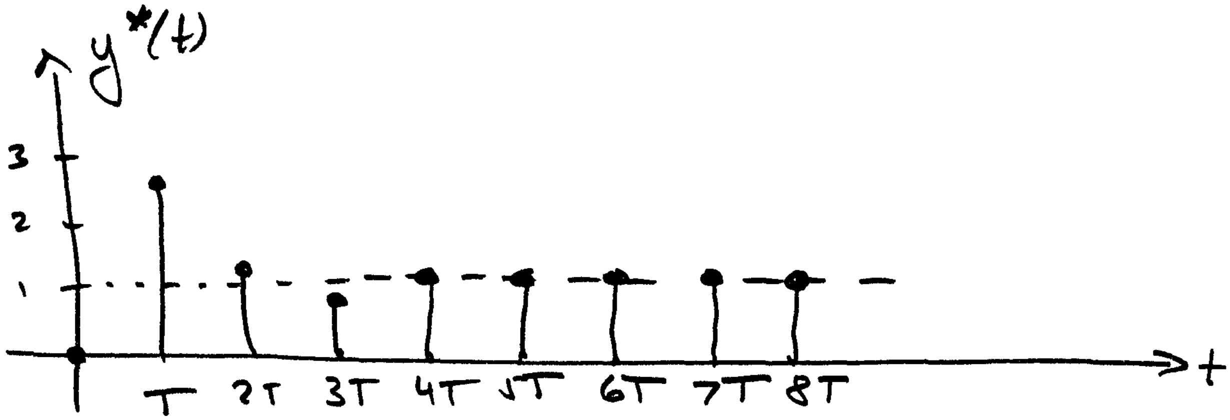
$$-\quad\frac{2}{3}z^{-3}-\frac{2}{3}z^{-4}$$

$$z^{-4}$$

$$-\quad z^{-4}-z^{-5}$$

$$z^{-5}\dots$$

$$\Rightarrow y(t)=\frac{8}{3}z^{-1}+\frac{4}{3}z^{-2}+\frac{2}{3}z^{-3}+z^{-4}+z^{-5}+z^{-6}+\dots$$



We recognize that:

(i) The response is by no means infinitely fast.
⇒ There is no reason to expect the $\underline{k}'$-values to grow beyond all bounds.

(ii) This system exhibits the <u>correct</u> final answer after 4 steps. Is this related to the fact that this is a fourth-order system?
⇒ This is no accident. Let me prove this in due course.

(iii) There is no reason to assume that the system doesn't exhibit a ripple after $t = 4T$. In all likelihood, this will be the case.

Let us consider the following step response:

$$y^*(t) = \{\phi, a_1, a_2, \dots, a_{n-1}, 1, 1, 1, \dots\}$$

$$\Rightarrow \mathcal{Y}(z^{-1}) = a_1 z^{-1} + a_2 z^{-2} + \dots + a_{n-1} z^{-(n-1)} + z^{-n} + z^{-(n+1)} + \dots$$

$$\Rightarrow z^{-1} \cdot \mathcal{Y}(z^{-1}) = a_1 z^{-2} + a_2 z^{-3} + \dots + a_{n-1} \cdot z^{-n} + z^{-(n+1)} + \dots$$

$$(1 - z^{-1}) \cdot \mathcal{Y}(z^{-1}) = a_1 z^{-1} + (a_2 - a_1) z^{-2} + (a_3 - a_2) z^{-3} + \dots + (a_{n-1} - a_{n-2}) z^{-(n-1)} + (1 - a_{n-1}) z^{-n}$$

$$\Rightarrow \mathcal{Y}(z^{-1}) = \frac{a_1 z^{-1} + (a_2 - a_1) z^{-2} + \dots + (a_{n-1} - a_{n-2}) z^{-(n-1)} + (1 - a_{n-1}) z^{-n}}{1 - z^{-1}}$$

$$\Rightarrow \mathcal{Y}(z) = \frac{a_1 \cdot z^{n-1} + (a_2 - a_1) z^{n-2} + \dots + (a_{n-1} - a_{n-2}) z + (1 - a_{n-1})}{z^{n-1} \cdot (z - 1)}$$

$$\Rightarrow \mathcal{G}_{tot}(z) = \frac{\mathcal{Y}(z)}{\mathcal{U}(z)} = \frac{\mathcal{P}_{tot}(z)}{z^n}$$

q.e.d.

## Conclusions:

1) Deadbeat response is achievable with finite resouces, whereas the corresponding continuous-system design is <u>not</u> practical.

2) Poles in the s-plane at $-\infty$ can be "spaced out," so that the sensitivity increase due to pole collocation is not an issue. This, however is a major issue for deadbeat design. Due to the multiple pole problem, the solution is highly sensitive to parameter variations, i.e., small changes in controller parameter values will place the closed-loop pole all over the z-plane.

3) Due to the inter-sampling ripple effect, the benefits of deadbeat control design may actually be less impressive than one would think.