On Properties of Random Triangulations

One of the most central questions in modern computer science is the analysis of algorithms. When analyzing an algorithm, two types of statements are of importance: the worst-case analysis, where the behaviour of the given algorithm is analysed for ill-behaved input instances, and the average-case analysis, where the average behaviour of the algorithm over all feasible inputs is considered. From a practical point of view, the average-case analysis is particularly important when the worst-case analysis does not result in satisfactory quality characteristics about the given algorithm: it is still possible, that the algorithm behaves well in real-world applications, although a bad worst-case behaviour can be mathematically proven. A typical example for this phenomenon is the well-known sorting algorithm QuickSort.

In order to study the average-case behaviour of an algorithm, we have to specify a probability distribution on the class of all inputs. In case of graph problems, a possible approach is to use the uniform distribution on the set of graphs on a given vertex set. For algorithms working on special graphs, e.g. on planar graphs, or graphs with other global structural restrictions, we might use the uniform distribution on the specific graph class.

Graphs with structural constraints, such as planar graphs or subclasses of them, have numerous applications in real world scenarios, for example chip manufacturing, drawing of diagrams, computer graphics, and many more. On the other hand, performing an average-case analysis of algorithms working on such graphs is presently a very difficult and challenging problem, as very little is known about “how” a “typical” graph in such a class looks like. It is for instance an open problem to determine the expected number of vertices of degree 4 in a random planar graph, or the expected maximum degree of a vertex, or the size of a maximum biconnected subgraph. These problems have attracted significant attention over the last years, and are the topic of several papers, see e.g. [1], [2], [3], and [4], and references therein.

In this project we want to investigate which tools might be suitable to tackle some of the above mentioned problems. Instead of looking at planar graphs, we will restrict ourselves to a “simpler” class, namely maximum planar graphs or triangulations. The main focus will be on determining the asymptotic behaviour of the degree sequence of a random triangulation on \( n \) labelled vertices, i.e., the expected number of vertices of degree 3,4,….. Moreover, if time permits, we want to examine if the developed techniques can be extended to answer related questions. In this context it would be interesting to investigate if the actual number of vertices of given degree takes values near the expectation with high probability, or to explore other parameters of random triangulations.

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References