Interactive Exercise "Optimal Binary Search Tree"

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2. The Frequency Matrix F

$$\begin{aligned}\mathbf{F}(i,j) &:= \mathbf{F}(i+1,j) + \mathbf{F}(i,i) \ , \\ \mathbf{F}(i,i) &:= f(i) \quad \text{(frequency of key with index } i) \ .\end{aligned}$$

Explanation: Start with element (i,j) and walk down the column recursively until the diagonal entry (j,j) is reached.

3. The Optimal Cost Matrix C

$$\mathbf{C}(i,j) := \mathbf{F}(i,j) + \min\Big\{ \overbrace{\mathbf{C}(i,j-1)}^{\text{left tree empty}} \ , \ \overbrace{\mathbf{C}(i+1,j)}^{\text{right tree empty}} \ , \ \underbrace{\min_{i+1\le k\le j-1} \mathbf{C}(i,k-1) + \mathbf{C}(k+1,j)}_{\substack{k\text{: root of optimal tree with nodes } i,\ldots,j \\ \text{optimal root index } k=\mathbf{R}(i,j) \text{ splits this tree} \\ \text{into the subtrees } i,\ldots,k-1 \text{ and } k+1,\ldots,j}} \Big\}$$

4. The Optimal Root Index Matrix R

$\mathbf{R}(i,j)$: Index of root of OBST with keys $i, i-1, \ldots, j+1, j$.
$\mathbf{R}(1,n)$: Index of root of OBST with all keys.