18-799F Algebraic Signal Processing Theory
Spring 2007
Assignment 7
Due Date: April 4th 2:30pm (at the beginning of class)

This is a Matlab assignment. Please turn in all code, plots (you can paste into word for example), and Matlab command line output if appropriate (also copy-paste).
In this assignment you will use a 1-D image grayscale image of size 512×1. The image values have 8 bits, i.e., they are in the range $0, \ldots, 255$. You can download file *image512.mat* from the course webpage. Load into Matlab with: load('image512.mat');
The image is stored in the variable *hw_image* and referred to as $\hat{s}$.

(a) Plot $\hat{s}$ as a function, i.e. pixel numbers on the x-axis and pixel values on the y-axis.

(b) Recall that the energy of a discrete signal is equal to the square of its ℓ^2 norm: $E(\hat{s}) = \|\hat{s}\|_2^2 = \sum_i s_i^2$.
Calculate the energy of $\hat{s}$. You can use Matlab's *norm* function.

(c) Write Matlab functions that compute for any input length n: a) the orthogonal DCT of type 2 and its inverse; b) the orthogonal DCT of type 4 and its inverse. Refer to the "Algebraic Theory of Signal Processing" paper for the exact form of these orthogonal transforms.

For size 512, for both transforms, compute the first (lowest) 8 pure frequencies (as column vectors, i.e., in coordinates) and plot them as functions.

(d) Compute the orthogonal DCT-2 and DCT-4 of $\hat{s}$, which we will call y_2 and y_4, and plot them as functions. Compute the energy of y_2 and y_4 and compare it to the energy of $\hat{s}$. Explain the result.

Which signal models $(\mathcal{A}, \mathcal{M}, \Phi)$ did you implicitly adopt by applying the DCT-2 and DCT-4?

(e) We define

$$R_{t,k} = \frac{\|y_t(1:k)\|_2^2}{\|y_t\|_2^2}, t = 2, 4.$$

In words, $R_{t,k}$ is the fraction of energy contained in the first k coefficients of the spectrum of the signal (where the notion of spectrum depends on the transform). Clearly, $R_{t,512} = 1$.

Plot $R_{t,k}$ as function of k for $t = 2, 4$.

For $t = 2, 4$, find the minimum values of k that satisfy, respectively, a) $R_{t,k} = 0.5$; b) $R_{t,k} = 0.9$; c) $R_{t,k} = 0.99$? In other words, how many coefficients are required to preserve a) 50%; b) 90%, and c) 99% of the energy?