How to Write Fast Code

18-645, spring 2008
18th Lecture, Mar. 24th

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Complexity of the DFT

- **Measure: L_c, $2 \le c$**
  - Complex adds count 1
  - Complex mults by a constant a with $|a| < c$ counts 1
  - L_2 is strictest, $L_{infinity}$ the loosest (and most natural)

- **$n = 2^k$: $L_2(DFT_n) \le 3/2\ n \log_2(n)$ (using Cooley-Tukey FFT)**
- **General n: $L_2(DFT_n) \le 8\ n \log_2(n)$ (needs Bluestein FFT)**
- **Theorem (Morgenstern): c < infinity**
  $L_c(DFT_n) \ge ½\ n \log_c(n)$
  Implies: in the measure L_c, the DFT is $\Theta(n \log(n))$

- **More details: Algebraic Complexity Theory**

The History of Fast Transform Algorithms

- **… starts with the FFT**

- **The advent of digital signal processing is often attributed to the FFT (Cooley-Tukey 1965)**

- **History:**
  - Around 1805: FFT discovered by Gauss [1]
    (Fourier publishes the concept of Fourier analysis in 1807!)
  - 1965: Rediscovered by Cooley-Tukey
  - 2002: James W. Cooley receives the IEEE Jack S. Kilby Signal Processing Medal "For pioneering the Fast Fourier Transform (FFT) algorithm."

[1]: Heideman, Johnson, Burrus: "Gauss and the History of the Fast Fourier Transform" Arch. Hist. Sc. 34(3) 1985

Carl-Friedrich Gauss

1777 - 1855

- **Contender for the greatest mathematician of all times**
- **Some contributions:** Modular arithmetic, least square analysis, normal distribution, fundamental theorem of algebra, Gauss elimination, Gauss quadrature, Gauss-Seidel, non-euclidean geometry, …