Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Polyhedral Computation, Spring 2014
Assignment 6*

May 13 , 2014

Problem 1 (Bimatrix Game):

Consider the bimatrix game given in terms of

$$A = \begin{pmatrix} 6 & 3 \\ 1 & 5 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 1 \\ 3 & 6 \end{pmatrix}.$$

1. Is there a pure Nash equilibrium?

2. Is the mixed strategy $((1/2, 1/2), (1/2, 1/2))$ a Nash equilibrium?

3. Depict the two associated polytopes

$$P_1 = \{x \in \mathbb{R}^2 : x \geq \mathbf{0}, B^T x \leq \mathbf{1}\} \text{ and}$$
$$P_2 = \{y \in \mathbb{R}^2 : Ay \leq \mathbf{1}, y \geq \mathbf{0}\}.$$

   Normalize all non-zero extreme points of P_1 and P_2. The arising vectors are denoted by P_1' and P_2' respectively. Build pairs of vectors, one in P_1' and the other one in P_2', to enumerate all Nash equilibria.

4. Use the program given in http://banach.lse.ac.uk/form.html to compute all Nash equlibria of the bimatrix game given by

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}.$$

*Please hand in your solution to May Szedlák CAB G19.2 (may.szedlak@inf.ethz.ch) no later than **Tuesday, May 27, 2014**.

Problem 2 (Central Arrangement): Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ -1 & 1 \end{pmatrix}$$

and let $\mathcal{A}$ be the central arrangement with representation $(A, \mathbf{0})$.

1. Draw the arrangement $\mathcal{A}$.
2. Compute the face lattice $\mathcal{F}(\mathcal{A})$.
3. Determine P_A and its dual $(P_A)^*$, and check Theorem 10.2.

Problem 3 (Face Lattice of Central Arrangements): Prove Theorem 10.2. Assume that $A \in \mathbb{R}^{m\times d}$ has full column rank, that is, $Ax = \mathbf{0}$ implies $x = \mathbf{0}$. Proceed as described in the following. For any $y \in \mathbb{R}^m$, let $\operatorname{sgn}(y)$ be the corresponding sign vector with $\operatorname{sgn}(y)_i$ equal to 1 if $y_i > 0$, -1 if $y_i < 0$ and 0 otherwise. Given any non-zero vector $x \in \mathbb{R}^d$, let $p(x) := \frac{x}{\operatorname{sgn}(Ax)^T Ax}$ and $a(x) := \{y \in \{1, -1\}^m : y^T Ax = 1\}$. Show that for every non-zero x and all $y \in \{1, -1\}^m$, we have

1. $p(x) \in P_A$, and
2. $y \in a(p(x))$ if and only if $\operatorname{sgn}(y)_i = \operatorname{sgn}(Ax)_i$ for all components i with $(Ax)_i \neq 0$.