Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Polyhedral Computation, Spring 2014
Assignment 5*

April 29 , 2014

Problem 1 (Feasible Region Half Integral):
Consider a simple graph $G(V, E)$. Then the relaxation formula of the Perfect Matching Problem is

$$\begin{aligned} &\max \sum_{e \in E} x_e \\ &s.t. \sum_{e \in \delta(v)} x_e = 1 \text{ for all } v \in V \\ &\qquad x_e \geq 0 \text{ for all } e \in E \end{aligned}$$

Prove that the feasible region is half-integral, that is, the components of every extreme point are $\{0, \frac{1}{2}, 1\}$.

Problem 2 (Polyhedral Computation Codes): Download the program `scdd_gmp` from http://www.inf.ethz.ch/personal/fukudak/cdd_home/index.html.

1. Run the program on the example given in the "Matching Polytope"' paper.
2. Add blossom inequalities to define the matching polytope. Which ones do we need?

Problem 3 (Redundant Inequalities): Consider a system $Ax \leq b$ of m linear inequalities in d variables. The system is called consistent if it has a solution, inconsistent otherwise. We define the *homogenization* as the system $Ax \leq bx_0$ and $x_0 \geq 0$ with one extra variable.

1. Let $Ax \leq b$ be consistent. Prove that an inequality $A_i x \leq b_i$ is redundant in $Ax \leq b$ if and only if the corresponding inequality $A_i x \leq b_i x_0$ is redundant in the homogenization.

*Please hand in your solution to May Szedlák CAB G19.2 (may.szedlak@inf.ethz.ch) no later than **Tuesday, May 13, 2014**.

2. Suppose $Ax \leq b$ is inconsistent. Find a counterexample to the statement in a)

Problem 4 (Redundancy): Prove Proposition 8.10.