Int. J. General Systems, Vol. 24(1–2), pp. 95–116
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COMBINED QUALITATIVE/QUANTITATIVE SIMULATION MODELS OF CONTINUOUS-TIME PROCESSES USING FUZZY INDUCTIVE REASONING TECHNIQUES*

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(Received August 24, 1993; in final form August 24, 1994)

A new mixed quantitative and qualitative simulation methodology based on fuzzy inductive reasoning is presented. The feasibility of this methodology is demonstrated by means of a simple hydraulic control system. The mechanical and electrical parts of the control system are modeled using differential equations, whereas the hydraulic part is modeled using fuzzy inductive reasoning. The technique is described in detail in the first part of this paper. The example is shown in the second part of the paper. The mixed quantitative and qualitative model is simulated in ACSL, and the simulation results are compared with those obtained from a fully quantitative model. The example was chosen as a simple to describe, yet numerically demanding process whose sole purpose is to prove the concept. Several practical applications of this mixed modeling technique are mentioned in the paper, but their realization has not yet been completed.

INDEX TERMS: Modeling, simulation, mixed quantitative and qualitative models, inductive reasoning, forecasting theory, fuzzy systems, learning systems, artificial intelligence

*This paper is based on an earlier conference version presented at the IFAC Symposium on Intelligent Components and Instruments for Control, Málaga, Spain, 1992

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1. INTRODUCTION

Qualitative simulation has recently become a fashionable branch of research in artificial intelligence. Human reasoning has been understood as a process of mental simulation, and qualitative simulation has been introduced as an attempt to replicate, in the computer, facets of human reasoning.

Qualitative simulation can be defined as evaluating the behavior of a system in qualitative terms [Cellier, 1991b]. To this end, the states that the system can be in are lumped together to a finite (discrete) set. For example, instead of dealing with temperature as a real-valued quantity with values such as 22.0°C, or 71.6°F, or 295.15 K, qualitative temperature values may be characterized as 'cold,' 'warm,' or 'hot.'

Qualitative variables are variables that assume qualitative values. Variables of a dynamical system are functions of time. The behavior of a dynamical system is a description of the values of its variables over time. The behavior of quantitative variables is usually referred to as trajectory behavior, whereas the behavior of qualitative variables is commonly referred to as episodical behavior. Qualitative simulation can thus be defined as a process of inferring the episodical behavior of a qualitative dynamical system or model.

Qualitative variables are frequently interpreted as an ordered set without distance measure [Babbie 1989]. It is correct that 'warm' is "larger" (warmer) than 'cold,' and that 'hot' is "larger" (warmer) than 'warm.' Yet, it is not true that

$$\text{'warm'} - \text{'cold'} = \text{'hot'} - \text{'warm'} \tag{1}$$

or, even more absurdly, that

$$\text{'hot'} = 2 * \text{'warm'} - \text{'cold'} \tag{2}$$

Operators such as '–' and '*' are not defined for qualitative variables.

Time, in a qualitative simulation, is also frequently treated as a qualitative variable. It is then possible to determine whether one event happens before or after another event, but it is not possible to specify when precisely a particular event takes place.

The most widely advocated among the qualitative simulation techniques are the knowledge-based approaches that were originally derived from the *Naïve Physics Manifesto* [Hayes 1979]. Several dialects of these types of qualitative models exist [de Kleer and Brown 1984; Forbus 1984; Kuipers 1986]. They are best summarized in Bobrow [1985].

The purpose of most qualitative simulation attempts is to enumerate, in qualitative terms, all possible episodical behaviors of a given system under all feasible experimental conditions. This is in direct contrast to quantitative simulations that usually content themselves with generating one single trajectory behavior of a given system under one single set of experimental conditions.

2. MIXED MODELS

In the light of what has been explained above, it seems questionable whether mixed quantitative and qualitative models are feasible at all. How should a mixed quantitative and qualitative simulation deal with the fact that the quantitative subsystems treat the independent variable, *time*, as a quantitative variable, whereas the qualitative subsystems treat the same variable qualitatively? When does a particular qualitative event occur in terms of quantitative time? How are the explicit experimental conditions that are needed by the quantitative subsystems accounted for in the qualitative subsystems?

Quite obviously, a number of incompatibility issues exist between quantitative and qualitative subsystems that must be settled before mixed simulations can be attempted. In a mixed simulation, also the qualitative subsystems must treat time as a quantitative variable. Furthermore, the purpose of qualitative models in the context of mixed simulations is revised. It is no longer their aim to enumerate episodical behaviors. Instead, also the qualitative models are now used to determine a single episodical behavior in response to a single set of qualitative experimental conditions.

Do so revised qualitative models make sense? It is certainly illegitimate to request that, because human aircraft pilots are unable to solve Riccati equations in their heads to determine an optimal flight path, autopilots shouldn't tackle this problem either. It is not sufficient to justify the existence of qualitative models by human inadequacies to deal with quantitative information.

Two good reasons for dealing with information in qualitative ways are the following:

1. Quantitative details about a (sub)system may not be available. For example, while the mechanical properties of a human heart are well understood and can easily be described by differential equation models, the effects of many chemical substances on the behavior of the heart are poorly understood and cannot easily be quantified. A mixed model could be used to describe those portions of the overall system that are well understood by quantitative differential equation models, while other aspects that are less well understood may still be representable in qualitative terms.
2. Quantitative details may limit the robustness of a (sub)system to react to previously unknown experimental conditions. For example, while a human pilot is unable to compute an optimal flight path, he or she can control the airplane in a much more robust fashion than any of today's autopilots. Optimality in behavior can be traded for robustness. A fuzzy controller is an example of a qualitative subsystem that is designed to deal with a larger class of experimental conditions in suboptimal ways. Mixed quantitative and qualitative models may be used to address either or both of the above applications. However, in order to do so, it is necessary to devise qualitative modeling and simulation capabilities that are compatible with their quantitative counterparts and that can be used to represent qualitative subsystems such as those mentioned above appropriately and in terms of knowledge available to the system designer at the time of modeling.

It is the purpose of this paper to describe one such mixed modeling and simulation methodology. In the advocated approach, the qualitative subsystems are represented (modeled) by a special class of finite state machines called fuzzy optimal masks, and their episodical behavior is inferred (simulated) by a technique called fuzzy forecasting. The overall process of qualitative modeling and simulation is referred to as fuzzy inductive reasoning.

Fuzzy inductive reasoning is accomplished using SAPS-II [Cellier and Yandell 1987], a software that evolved from the General System Problem Solving (GSPS) framework [Klir 1985, 1989; Uyttenhove 1979]. SAPS-II is implemented as a function library of CTRL-C [Systems Control Technology, 1985]. A subset of the SAPS-II modules, namely the recoding, forecasting, and signal regeneration modules have also been made available as an application library of ACSL [Mitchell & Gauthier 1991], which is the software used in the mixed quantitative and qualitative simulation runs.

3. FUZZY INDUCTIVE REASONING

3.1. Fuzzy Recoding

Recoding denotes the process of converting a quantitative variable to a qualitative variable. In general, some information is lost in the process of recoding. Obviously, a temperature value of 97°F contains more information than the value 'hot.' Fuzzy recoding avoids this problem.

Figure 1 shows the fuzzy recoding of a variable called "systolic blood pressure." For example, a quantitative systolic blood pressure of 135.0 is recoded into a qualitative value of 'normal' with a fuzzy membership function of 0.895 and a side function of 'right.' Thus, a single quantitative value is recoded into a triple. Any systolic blood pressure with a quantitative value between 100.0 and 150.0 will be recoded into the qualitative value 'normal.' The fuzzy membership function denotes the value of the bell-shaped curve shown in Figure 1, always a value between 0.5 and 1.0. It was decided to use bell-shaped fuzzy membership functions rather than the more commonly used triangular ones. This membership function can be easily calculated using the equation:

$$Memb_i = \exp(-\tau_i \cdot (x - \mu_i)^2) \tag{3}$$

where x is the continuous variable to be recoded, μ_i is the algebraic mean between two neighboring landmarks, and τ_i is determined such that the membership function, $Memb_i$, degrades to a value of 0.5 at the landmarks. Contrary to other fuzzy approaches, the tails of the membership functions ($Memb_i < 0.5$) are ignored in the method described in this paper. The decision to ignore the tails of the membership functions is related to the selection of the fuzzy inferencing technique, and is justified in Mugica and Cellier [1993].

The side function indicates whether the quantitative value is to the left or to the right of the maximum of the fuzzy membership function. Obviously, the qualitative triple contains the same information as the original quantitative variable. The quantitative value

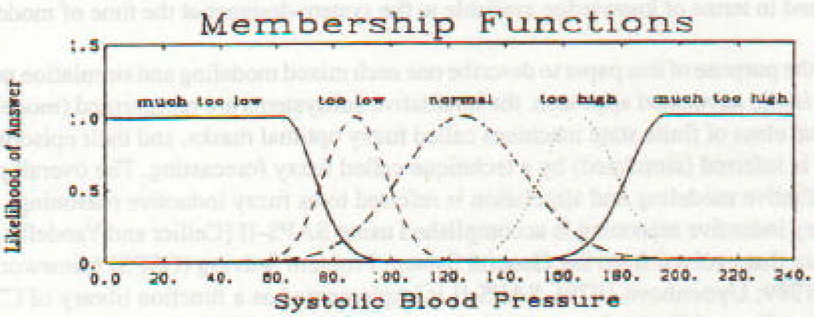

Figure 1 Fuzzy recoding.

can be regenerated accurately, i.e., without any loss of information, from the qualitative triple.

At this point, the question can be raised, how many discrete levels should be selected for each state variable, and where the borderlines (landmarks) that separate two neighboring regions from each other are to be drawn.

From statistical considerations, it is known that in any class analysis, each legal discrete state should be recorded at least five times [Law and Kelton 1990]. Thus, a relation exists between the total number of legal states and the number of data points required to base the modeling effort upon:

$$n_{rec} \geq 5 \cdot n_{leg} = 5 \cdot \prod_{\forall i} k_i \tag{4}$$

where n_{rec} denotes the total number of recordings, i.e., the total number of observed states, n_{leg} denotes the total number of distinct legal states, i is an index that loops over all variables in the state, and k_i denotes the number of levels that the i^{th} variable can assume. The number of variables is usually given, and the number of recordings is frequently predetermined. In such a case, the optimum number of levels, n_{lev}, of all variables can be found from the following equation:

$$n_{lev} = \text{ROUND}\left(\sqrt[n_{var}]{n_{rec}/5}\right) \tag{5}$$

assuming that all variables are classified into the same number of levels. For reasons of symmetry, an odd number of levels is often preferred over an even number of levels. Abnormal states ('too low,' 'too high,' and 'much too low,' 'much too high') are grouped symmetrically about the 'normal' state.

The number of levels chosen for each variable is very important for several reasons. This number influences directly the computational complexity of the inference stage. Traditional fuzzy systems usually require between seven and 13 classes for each variable [Aliev, *et al.* 1992; Maiers and Sherif 1985]. An exhaustive search in such a high-dimensional discrete search space would be very expensive, and the number of classes should therefore be reduced, if possible, to help speed up the optimization. It was shown in Mugica and Cellier [1993] that the selected fuzzy inferencing technique makes it possible to reduce the number of levels down to usually three or five, a number confirmed by several practical applications [Albornoz and Cellier 1993a, 1993b; Cellier 1991c; Vesanterä and Cellier 1989].

Once the number of levels of each variable has been selected, the landmarks must be chosen to separate neighboring regions from each other. There are several ways to find a meaningful set of landmarks. The most effective way is based on the idea that the expressiveness (or information contents) of the model will be maximized if each level is observed equally often. In order to distribute the observed trajectory values of each variable equally among the various levels, they are sorted into ascending order, the sorted vector is then split into n_{lev} segments of equal length, and the landmarks are chosen anywhere between the extreme values of neighboring segments, e.g., using the arithmetic mean values of neighboring observed data points in different segments.

3.2. *Fuzzy Optimal Masks*

By now, the quantitative trajectory behavior has been recoded into a qualitative episodical behavior. In SAPS-II, the episodical behavior is stored in a *raw data matrix*. Each column of the raw data matrix represents one of the observed variables, and each row of the raw data matrix represents one time point, i.e., one recording of all variables or one recorded state. The values of the raw data matrix are in the set of legal levels that the variables can assume, that means, they are all positive integers, usually in the range from '1' to '5' (SAPS-II uses integers in place of symbolic values to represent qualitative levels).

Masks as Qualitative Models. How does the episodical behavior support the identification of a model of a given system for the purpose of forecasting the future behavior for any given input stream?

A continuous trajectory behavior has been recorded and is available for modeling. It is further assumed that the inputs into the real system and the outputs that can be measured are known. The trajectory behavior can thus be separated into a set of input trajectories, u_i, concatenated from the right with a set of output trajectories, y_i, as shown in the following example containing two inputs and three outputs:

$$\begin{array}{l} \textit{time} \\ 0.0 \\ \delta t \\ 2\cdot\delta t \\ 3\cdot\delta t \\ \vdots \\ (n_{rec}-1)\cdot\delta t \end{array} \quad \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} \end{array} \tag{6}$$

In order to avoid possible ambiguities, it is defined that the terms "inputs" and "outputs," when used in this paper without further qualifier, shall always refer to the input and output variables of the subsystem to be modeled by the qualitative reasoner.

In the process of modeling, it is desired to discover finite automata relations among the recoded variables that make the resulting state transition matrices as deterministic as possible. If such a relationship is found for every output variable, the behavior of the system can be forecast by iterating through the state transition matrices. The more deterministic the state transition matrices are, the better the certainty that the future behavior will be predicted correctly.

A possible relation among the qualitative variables for this example could be of the form:

$$y_1(t) = \tilde{f}(y_3(t-2\delta t),\ u_2(t-\delta t),\ y_1(t-\delta t),\ u_1(t)) \tag{7}$$

Equation (7) can be represented as follows:

$$\begin{array}{c} {}_t\backslash^x \\ t-2\delta t \\ t-\delta t \\ t \end{array} \quad \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & -2 & -3 & 0 & 0 \\ -4 & 0 & +1 & 0 & 0 \end{pmatrix} \end{array} \tag{8}$$

The negative elements in this matrix are referred to as *m*-inputs. *M*-inputs denote inputs of the qualitative functional relationship. They can be either inputs or outputs of the subsystem to be modeled, and they can represent different time instants. The above example contains four m-inputs. The sequence in which they are enumerated is immaterial. They are usually enumerated from left to right and top to bottom. The positive value denotes the m-output. In the above example, the first m-input, i_1, corresponds to the output variable y_3 two sampling intervals back: $y_3(t - 2\delta t)$, whereas the second m-input refers to the input variable u_2 one sampling interval in the past: $u_2(t - \delta t)$, etc.

In inductive reasoning, such a representation is called a *mask*. A mask denotes a dynamic relationship among qualitative variables. A mask has the same number of columns as the episodical behavior to which it should be applied and it has a certain number of rows. The number of rows of the mask matrix is called the *depth* of the mask.

The mask can be used to flatten a dynamic relationship out into a static relationship. It can be shifted over the raw data matrix, the selected m-inputs and m-output can be extracted from the raw data, and they can be written next to each other in one row of the so-called *input/output matrix*. Figure 2 illustrates this process. After the mask has been applied to the raw data, the formerly dynamic episodical behavior has become static, i.e., the relationships are now contained within single rows.

Each row of the input/output matrix is called a *state* of the system. The state consists of an *input state* and an *output state*. The input state denotes the vector of values of all the m-inputs belonging to the state, and the output state is the value of the single m-output of the state. The set of all possible states is referred to as the set of *legal states* of the qualitative model.

It has not been discussed yet how the time distance between two logged entries of the trajectory behavior, δt, is chosen in practice. In a combined quantitative/qualitative simulation (mixed simulation), δt must be selected carefully because its value will strongly influence the mask selection process. Determining a good value for this parameter in a systematic way is currently the object of intensive research. In general, experience has shown that the mask should cover the largest time constant that has to be captured in the model.

If the trajectory behavior stems from measurement data, a Bode diagram of the system to be modeled should be made. This enables to determine the eigenfrequencies of the system, and in particular, the smallest and largest eigenfrequencies. The smallest eigenfre-

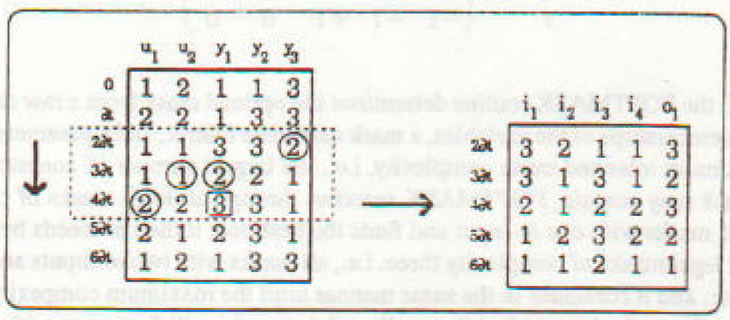

Figure 2 Flattening dynamic relationships through masking.

quency ω_{low} is the smallest frequency, at which the tangential behavior of the amplitude of the Bode diagram changes by –20 dB/decade, and the largest eigenvalue ω_{high} is the highest frequency where this happens. The largest time constant, $t_{settling}$, and the shortest time constant, t_{fast}, of the system can then be computed as follows:

$$t_{settling} = \frac{2\pi}{\omega_{low}};\ t_{fast} = \frac{2\pi}{\omega_{high}} \tag{9}$$

The mask depth should be chosen as an integer approximation of the ratio between the largest and smallest time constants to be captured in the model plus one:

$$depth = \mathrm{INT}\left(\frac{t_{settling}}{t_{fast}}\right) + 1 \tag{10}$$

but this ratio should not be much larger than 3 or 4. Otherwise, the inductive reasoner won't work very well, since the computing effort grows factorially with the size of the mask. Multiple frequency resolution in inductive reasoning is still an area of open research.

If the chosen mask depth is 3, the mask spans a time interval of $2\delta t$, thus:

$$\delta t = \frac{t_{settling}}{2} \tag{11}$$

Finding the Optimal Mask How is a mask found that, within the framework of all allowable masks, represents the most deterministic state transition matrix? This mask will optimize the predictiveness of the model. In SAPS-II, the concept of a *mask candidate matrix* has been introduced. A mask candidate matrix is an ensemble of all possible masks from which the best is chosen by a mechanism of exhaustive search. The mask candidate matrix contains (–1) elements where the mask has a potential m-input, a (+1) element where the mask has its m-output, and (0) elements to denote forbidden connections. Thus, a good mask candidate matrix for the previously introduced five-variable example might be:

$$\begin{array}{c} {}_{t}\backslash^{x} \\ t-2\delta t \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & 0 & 0 \end{pmatrix} \end{array} \tag{12}$$

In SAPS-II, the FOPTMASK routine determines the optimal mask from a raw data matrix, the fuzzy memberships of the variables, a mask candidate matrix, and a parameter that limits the maximum tolerated mask complexity, i.e., the largest number of nonzero elements that the mask may contain. FOPTMASK searches through all legal masks of complexity two, i.e., all masks with one *m*-input and finds the best one; it then proceeds by searching through all legal masks of complexity three, i.e., all masks with two *m*-inputs and finds the best of those; and it continues in the same manner until the maximum compexity has been reached. In all practical examples, the quality of the masks will first grow with increasing complexity, then reach a maximum, and then decay rapidly. A good value for the maximum complexity is usually five or six.

Each of the possible masks is compared to the others with respect to its potential merit. The optimality of the mask is evaluated with respect to the maximization of its forecasting power.

The Shannon entropy measure is used to determine the uncertainty associated with forecasting a particular output state given any legal input state. The Shannon entropy relative to one input state is calculated from the equation

$$H_i = \sum_{\forall o} p(o|i) \cdot \log_2 p(o|i) \tag{13}$$

where $p(o|i)$ is the conditional probability of a certain output state o to occur, given that the input state i has already occurred. The term probability is meant in a statistical rather than in a true probabilistic sense. It denotes the quotient of the observed frequency of a particular state divided by the highest possible frequency of that state.

The overall entropy of the mask is then calculed as the sum

$$H_m = -\sum_{\forall i} p(i) \cdot H_i \tag{14}$$

where $p(i)$ is the probability of that input state to occur. The highest possible entropy H_{max} is obtained when all probabilities are equal, and a zero entropy is encountered for relationships that are totally deterministic.

A normalized overall entropy reduction H_r is defined as

$$H_r = 1.0 - \frac{H_m}{H_{max}} \tag{15}$$

H_r is obviously a real number in the range between 0.0 and 1.0, where higher values usually indicate an improved forcasting power. The optimal mask among a set of mask candidates is defined as the one with the highest entropy reduction.

The fuzzy membership associated with the value of a qualitative variable is a *measure of confidence*. In the computation of the input/output matrix, a confidence value can be assigned to each row. The confidence of a row of the input/output matrix is the joint membership of all the variables associated with that row [Li and Cellier 1990].

The joint membership of i membership functions is defined as the smallest individual membership:

$$Memb_{joint} = \bigcap_{\forall i} Memb_i = \inf_{\forall i}(Memb_i) \overset{def}{=} \min_{\forall i}(Memb_i) \tag{16}$$

The confidence vector indicates how much confidence can be expressed in the individual rows of the input/output matrix.

The *basic behavior* of the input/output model can now be computed. It is defined as an ordered set of all observed distinct states together with a measure of confidence of each state. Rather than counting the observation frequencies (as would be done in the case of a probabilistic measure), the individual confidences of each observed state are accumulated. If a state has been observed more than once, more and more confidence can be expressed in it. Thus, the individual confidences of each observation of a given state are simply added together.

In order to be able to still use the Shannon entropy, which is a *probabilistic* measure of information content, in the computation of the fuzzy optimal mask, the accumulated confidences must first be converted back to values that can be interpreted as conditional probabilities. To this end, the confidences of all states containing the same input state are added together, and the confidence of each of them is then divided by this sum. The resulting normalized confidences can be interpreted as conditional probabilities.

Application of the Shannon entropy to a confidence measure is a somewhat questionable undertaking on theoretical grounds, since the Shannon entropy was derived in the context of probabilistic measures only. For this reason, some scientists prefer to replace the Shannon entropy by other types of performance indices [Klir, 1989; Shafer, 1976], which have been derived in the context of the particular measure chosen. However, from a practical point of view, numerous simulation experiments have shown that the Shannon entropy works satisfactorily also in the context of fuzzy measures.

One problem still remains. The size of the input/output matrix increases as the complexity of the mask grows, and consequently, the number of legal states of the model grows quickly. Since the total number of observed states remains constant, the frequency of observation of each state shrinks rapidly, and so does the predictiveness of the model. The entropy reduction measure does not account for this problem. With increasing complexity, H_r simply keeps growing. Very soon, a situation is encountered where every state that has ever been observed has been observed precisely once. This obviously leads to a totally deterministic state transition matrix, and H_r assumes a value of 1.0. Yet the predictiveness of the model will be dismal, since in all likelihood already the next predicted state has never before been observed, and that means the end of forecasting. Therefore, this consideration must be included in the overall quality measure.

It was mentioned earlier that, from a statistical point of view, every state should be observed at least five times [Law and Kelton, 1990]. Therefore, an *observation ratio*, O_r, is introduced as an additional contributor to the overall quality measure [Li and Cellier, 1990]:

$$O_r = \frac{5 \cdot n_{5\times} + 4 \cdot n_{4\times} + 3 \cdot n_{3\times} + 2 \cdot n_{2\times} + n_{1\times}}{5 \cdot n_{\text{leg}}} \tag{17}$$

where:

n_{leg} = number of legal input states;
$n_{1\times}$ = number of input states observed only once;
$n_{2\times}$ = number of input states observed twice;
$n_{3\times}$ = number of input states observed thrice;
$n_{4\times}$ = number of input states observed four times;
$n_{5\times}$ = number of input states observed five times or more.

If every legal input state has been observed at least five times, O_r is equal to 1.0. If no input state has been observed at all (no data are available), O_r is equal to 0.0. Thus, O_r can also be used as a quality measure.

The overall *quality of a mask*, Q_m, is then defined as the product of its uncertainty reduction measure, H_r, and its observation ratio, O_r:

$$Q_m = H_r \cdot O_r \tag{18}$$

The *optimal mask* is the mask with the largest Q_m value.

In SAPS-II, the FOPTMASK function returns the overall best mask found in the optimization; a row vector that contains the Shannon entropies of the best masks for every considered complexity, H_m; another row vector containing the corresponding uncertainty reduction measures, H_r; and yet another row vector listing the quality measures, Q_m, of these suboptimal masks. Finally, FOPTMASK also returns the *mask history matrix*, a matrix that consists of a horizontal concatenation of all suboptimal masks. One of these masks is the optimal mask, which, for reasons of convenience, is also returned separately.

3.3. Fuzzy Forecasting

Once the optimal mask has been determined, it can be applied to the given raw data matrix resulting in a particular input/output matrix. Since the input/output matrix contains functional relationships within single rows, the rows of the input/output matrix can now be sorted in alphanumerical order. The result of this operation is called the *behavior matrix* of the system. The behavior matrix is a finite state machine. For each input state, it shows which output is most likely to be observed.

Forecasting has now become a straightforward procedure. The mask is simply shifted further down beyond the end of the raw data matrix, the values of the *m*-inputs are read out from the mask, and the behavior matrix is used to determine the future value of the *m*-output, which can then be copied back into the raw data matrix. In fuzzy forecasting, it is essential that, together with the qualitative output, also a fuzzy membership value and a side value are forecast. Thus, fuzzy forecasting predicts an entire qualitative triple from which a quantitative variable can be regenerated whenever needed.

In fuzzy forecasting, the membership and side functions of the new input state are compared with those of all previous recordings of the same input state contained in the behavior matrix. The one input state with the most similar membership and side functions is identified. For this purpose, a normalized quantitative signal

$$d_i = class_i + side_i * (1 - Memb_i) \tag{19}$$

is computed for every element of the new input state, and the regenerated d_i values are stored in a vector. This reconstruction is then repeated for all previous recordings of the same input state. Finally, the L_2 norms of the difference between the d vector of the new input state and the d vectors of all previous recordings of the same input state are computed, and the previous recording with the smallest L_2 norm is identified. Its *output* and *side* values are then used as forecasts for the *output* and *side* values of the current state.

Forecasting of the new membership function is done a little differently. Here, the five previous recordings with the smallest L_2 norms are used (if at least five such recordings are found in the behavior matrix), and a distance-weighted average of their fuzzy membership functions is computed and used as the forecast for the fuzzy membership function of the current state.

Absolute weights are computed as follows:

$$w_{abs_k} = \frac{d_{max} - d_k}{d_{max}} \tag{20}$$

where the index k loops over the five closest neighbors, and $d_i \leq d_j$, $i < j$; $d_{max} = d_5$. The absolute weights are numbers between 0.0 and 1.0. Using the sum of the five absolute weights:

$$s_w = \sum_{\forall k} w_{abs_k} \tag{21}$$

it is possible to compute relative weights:

$$w_{rel_k} = \frac{w_{abs_k}}{s_w} \tag{22}$$

Also the relative weights are numbers between 0.0 and 1.0. However, their sum is always equal to 1.0. It is therefore possible to interpret the relative weights as percentages. Using this idea, the membership function of the new output can be computed as a weighted sum of the membership functions of the outputs of the previously observed five nearest neighbors:

$$Memb_{out_{new}} = \sum_{\forall k} w_{rel_k} \cdot Memb_{out_k} \tag{23}$$

The fuzzy forecasting function will usually give a more accurate forecast than the probabilistic forecasting function. A comparative study of the most commonly used inferencing methods and the five-nearest-neighbors defuzzification method is presented in [Mugica and Cellier, 1993]. This method allows to retrieve pseudo-continuous output signals with a high quality using the REGENERATE function. This means that also a forecast of the continuous-time signals can be obtained [Cellier 1991a]. Notice that the REGENERATE function is the inverse process of the RECODE function.

4. AN EXAMPLE

In the remainder of this paper, an example will be presented that demonstrates, for the first time, the process of mixed quantitative and qualitative simulation using fuzzy inductive reasoning. The example was chosen simple enough to be presented in full, yet complex enough to demonstrate the generality and validity of the approach. However, it is not suggested that the chosen example represent a meaningful application of mixed quantitative and qualitative simulation. The example was chosen to prove the concept and to clearly present the methodology, not as a realistic and meaningful application of the proposed technique.

Figure 3 shows a hydraulic motor with a four-way servo valve. The flows from the high-pressure line into the servo valve and from the servo valve back into the low-pressure line are turbulent. Consequently, the relation between flow and pressure is quadratic

$$\begin{aligned} q_1 &= k(x_0 + x)\sqrt{P_S - p_1} \\ q_2 &= k(x_0 - x)\sqrt{p_1 - P_0} \\ q_3 &= k(x_0 + x)\sqrt{p_2 - P_0} \\ q_4 &= k(x_0 - x)\sqrt{P_S - p_2} \end{aligned} \tag{24}$$

The chosen parameter values are $P_S = 0.137 \times 10^8$ N m^{-2}, $P_0 = 1.0132 \times 10^5$ N m^{-2}, $x_0 = 0.05$ m, and $k = 0.248 \times 10^{-6}$ kg$^{-1/2}$ m$^{3/2}$.

The change in the chamber pressures is proportional to the effective flows in the two chambers

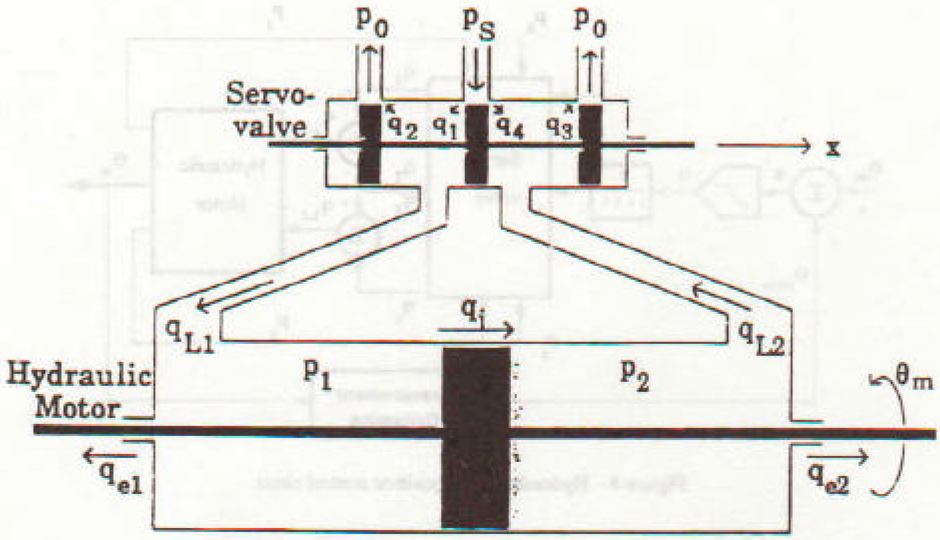

Figure 3 Hydraulic motor with a four-way servo valve.

$$\begin{aligned} \dot{p}_1 &= c_1(q_{L1} - q_i - q_{e1} - q_{ind}) \\ \dot{p}_2 &= c_1(q_{ind} + q_i - q_{e2} - q_{L2}) \end{aligned} \tag{25}$$

with $c_1 = 5.857 \times 10^{13}$ kg m^{-4} sec^{-2}. The internal leakage flow, q_i, and the external leakage flows, q_{e1} and q_{e2}, can be computed as

$$\begin{aligned} q_i &= c_i \cdot p_L = c_i(p_1 - p_2) \\ q_{e1} &= c_e \cdot p_1 \\ q_{e2} &= c_e \cdot p_2 \end{aligned} \tag{26}$$

where $c_i = 0.737 \times 10^{-13}$ kg^{-1} m^4 sec, and $c_e = 0.737 \times 10^{-12}$ kg^{-1} m^4 sec.

The induced leakage, q_{ind}, is proportional to the angular velocity of the hydraulic motor, ω_m

$$q_{ind} = \psi \cdot \omega_m \tag{27}$$

with $\psi = 0.575 \times 10^{-5}$ m^3, and the torque produced by the hydraulic motor is proportional to the load pressure, p_L

$$T_m = \psi \cdot p_L = \psi(p_1 - p_2) \tag{28}$$

The mechanical side of the motor has an inertia, J_m, of 0.08 kg m^2, and a viscous friction, ρ, of 1.5 kg m^2 sec^{-1}.

The hydraulic motor is embedded in the control circuitry shown on Figure 4. In the mixed quantitative and qualitative simulation, the mechanical and electrical parts of the control system will be represented by differential equation models, whereas the hydraulic part will be represented by a fuzzy inductive reasoning model.

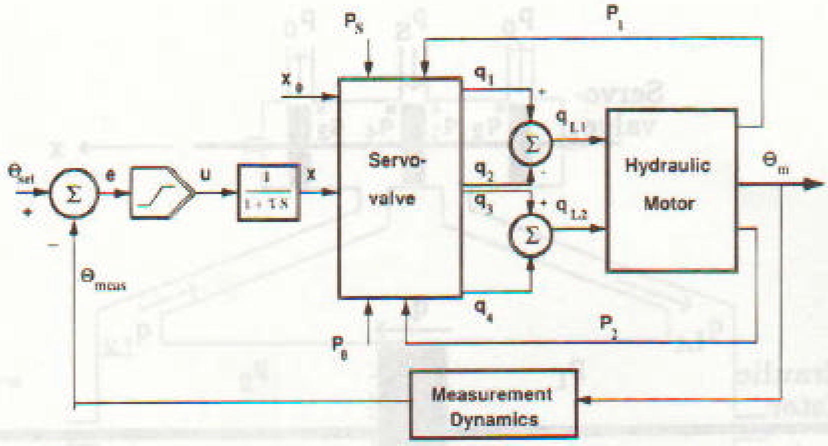

Figure 4 Hydraulic motor position control ciruit.

For this purpose, it was assumed that no knowledge exists that would permit a description of the hydraulic equations by means of a differential equation model. All that is known is that the mechanical torque, T_m, of the hydraulic motor somehow depends on the control signal, u, and the angular velocity, ω_m.

For validation purposes, the mixed simulation results will be compared with previously obtained purely quantitative simulation results. The purely quantitative simulation of the overall system (Fig. 4) was implemented by an ACSL program [Mitchell & Gauther 1991] simulated over 2.5 seconds. A binary random input signal was applied to the input of the system, θ_{set}.

The values of the control signal, u, the angular velocity, $\omega_m = \dot{\theta}_m$, and the torque, T_m, from the first 2.25 seconds of the quantitative simulation were recoded to generate the fuzzy inductive model of the hydraulic motor.

The values of the last 0.25 seconds of quantitative simulation were stored for validation purposes. Validation is accomplished by comparing the simulation results of the new mixed model with those of the purely quantitative model, which is being used in place of "measurement data."

4.1. Building the Fuzzy Inductive Model

As was described in the previous sections, the fuzzy inductive model is constructed in two steps. In the first step, the quantitative data are recoded, and in the second step, the fuzzy optimal mask is determined from the recoded data.

Fuzzy Recoding of the Hydraulics The first question to be addressed in the recoding process is the selection of the appropriate sampling rate (communication interval) for the continuous variables to be recorded (either from measurements or, as in this example, from a quantitative simulation study). In the given example, this value can be deduced from the longest time constant to be considered (i.e., the inverse of the slowest eigenvalue of the Jacobian). The eigenvalue is at −20, and therefore, the longest time constant is 0.05 seconds. In accordance with equation (10), the three variables u, ω_m, and T_m must thus be sampled once every 0.025 seconds if a mask depth of 3 is chosen [Cellier 1991a; Li and Cellier 1990].

Unfortunately, fuzzy inductive forecasting predicts only one value of T_m per sampling interval. Thus, the mixed qualitative and quantitative simulation behaves like a sampled-data control system with a sampling rate of 0.025. Thereby, the stability of the control system is lost because the sampling rate is too slow to keep up with the changes in the system. From a control system perspective, it is necessary to sample the variables considerably faster. An ACSL program was coded to study different sampling rates in order to obtain a stable control performance. This program introduces into the quantitative simulation a delay in the computation of the torque. The largest delay time that can be introduced without losing stability of the control system was identified. It was determined that the longest tolerable delay is 0.0025 seconds. Consequently, the mask depth must be increased from 3 to 21.

The next step is to find the number of discrete levels into which each of these variables will be recoded. For the given example, it was decided that all three variables can be sufficiently well characterized by three levels. A discretization of the variables in this manner implies that the number of legal states is 27 ($3 \times 3 \times 3$).

As explained before, it is desirable to record each state at least 5 times. Consequently, a minimum of 130 recordings, corresponding to a total simulation time of 0.325 seconds, is

```
//  Recode the system in an optimal manner

//  Start by sorting the observed trajectory values into ascending order.

meas = y(1:1001,2:4);
m = meas;
FOR i=1:3, ...
  [indx,mi] = SORT(meas(:,i)); ...
  m(:,i) = mi; ...
END

//  Cut the sorted vector into n_lev segments of equal length, and choose
//  the landmarks between the extreme values of neighboring segments.

LM = [ m(1,:)
       0.5*(m(333,:) + m(334,:))
       0.5*(m(666,:) + m(667,:))
       m(1001,:) ];
rawl = meas;
Membl = ONES(meas);
sidel = ZROW(meas);

//  Recode the observed trajectory values.

to = 1:3;
FOR i=1:3, ...
  from = [ LM(1:3,i) , LM(2:4,i) ]'; ...
  [r,m,s] = RECODE(meas(:,i),'fuzzy',from,to); ...
  rawl(:,i) = r;  Membl(:,i) = m;  sidel(:,i) = s; ...
END
```

Figure 5 Fuzzy recoding of hydraulic subsystem.

needed. However, due to the mismatch between the sampling rate required by fuzzy forecasting and the actually used sampling rate that is required due to the controller characteristics, considerably more data are needed. It was decided to choose a total simulation time of 2.5 seconds with 2.25 seconds being used for model identification, and the last 0.25 seconds being used for validation. This provides the optimal mask module with 900 recordings used for model identification, while fuzzy forecasting is carried out over the final 100 steps.

The fuzzy recoding is obtained using a CTRL-C program that invokes calls to the SAPS-II library described in subsection 3.1. It uses the previously recorded data from the purely quantitative simulation program. The CTRL-C program is shown on Figure 5.

Fuzzy Optimal Mask of the Hydraulics With the data recoded as was described above, it is possible to build the qualitative model of the hydraulics by means of the fuzzy optimal mask synthesis (subsection 2.2). To combine the qualitative and quantitative simulation models, it was necessary to solve the dynamic stability problem, while covering the longest time constant to be captured in the qualitative model. This means that, as mandated by control theory, the sampling interval δt is chosen to be 0.0025 seconds. Consequently, the mask depth must be chosen equal to 21. Even a search through all possible masks of complexities up to six only would be painfully slow. Therefore, the following approach was taken. From the point of view of fuzzy reasoning, a mask depth of three is usually sufficient. Consequently, it was decided to consider only inputs in the first, the 11th, and the 21st row of the mask, blocking all other rows out by setting the corresponding elements of the mask candidate matrix to 0. In this way, the search can proceed quickly, and yet, the resulting "optimal" mask will still be very close to the truly optimal mask. Thus, the following mask candidate matrix of depth 21 was chosen:

$$
\begin{array}{l} {}^{t}\backslash^{x} \\ t-20\delta t \\ t-19\delta t \\ \vdots \\ t-11\delta t \\ t-10\delta t \\ t-9\delta t \\ \vdots \\ t-\delta t \\ t \end{array}
\begin{array}{c} \begin{array}{ccc} u & \omega_m & T_m \end{array} \\ \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -1 & -1 & +1 \end{pmatrix} \end{array}
\qquad (29)
$$

Thus, the mechanical torque, T_m, at time t may depend on the current values of u and ω_m, as well as on past values of u, ω_m, and T_m at times $t - 0.025$ seconds and $t - 0.05$ seconds.

The fuzzy optimal mask is obtained by another CTRL-C program that also issues a call to the SAPS-II library. It makes use of the previously recoded data. This CTRL-C program is presented on Figure 6.

The following optimal mask has been found for this example:

```
// Perform an optimal mask analysis

// Extract the first 900 rows for model identification.

rraw = rawl(1:900,:);
MMemb = Membl(1:900,:);
sside = sidel(1:900,:);

// Select the mask candidate matrix.

mcan = ZROW(21,3);
mcan(1:10:21,:) = -ONES(3,3);
mcan(21,3) = 1;

// Determine the optimal mask.

[mask,HM,HR,Q,mhis] = FOPTMASK(rraw,MMemb,mcan,6)
```

Figure 6 Finding the optimal mask of the hydraulic subsystem.

$$\begin{array}{c} {}_t\backslash^x \\ t-20\delta t \\ t-19\delta t \\ \vdots \\ t-11\delta t \\ t-10\delta t \\ t-9\delta t \\ \vdots \\ t-\delta t \\ t \end{array} \begin{array}{c} \begin{array}{ccc} u & \omega_m & T_m \end{array} \\ \begin{pmatrix} 0 & -1 & -2 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ -3 & 0 & +1 \end{pmatrix} \end{array} \tag{30}$$

In other words:

$$T_m(t) = \tilde{f}(\omega_m(t-0.05),\ T_m(t-0.05),\ u(t)) \tag{31}$$

Fuzzy Forcasting and Signal Regeneration. Once the optimal mask has been determined and before it can be integrated into the mixed simulation, its prediction capability must be checked. For this purpose, another CTRL-C program was written that compares the values of T_m obtained from the quantitative simulation with the forecast and regenerated values. As mentioned before, the first 900 rows of the raw data matrix were used as past history data to compute the optimal mask. Fuzzy forecasting (subsection 3.3) is being used to predict new qualitative triples for T_m, but only for the last 100 rows of the raw data matrix. From the predicted qualitative triples, quantitative values can then be regenerated. This CTRL-C program is shown on Figure 7.

Figure 8 compares the true "measured" values of T_m obtained from the purely quantitative simulation (solid line) with the forecast and regenerated values obtained from the fuzzy inductive reasoning (dashed line). The results are encouraging. Quite obviously, the opti-

```
// Forecast the system over 100 steps

// Copy the quantitative data over.

rraw = raw1(1:1000,:);
MMemb = Memb1(1:1000,:);
sside = side1(1:1000,:);

// Destroy the last 100 rows.

rraw(901:1000,3) = ZROW(100,1);
MMemb(901:1000,3) = 0.75*ONES(100,1);
sside(901:1000,3) = ONES(100,1);

// Forecast new values for the last 100 rows.

[frcst,Mfrcst,sfrcst] = FFORECAST(rraw,Mmemb,sside,mask,900);

// Extract the forecast data.

frcdat = frcst(901:1000,3);
Mfrcdat = Mfrcst(901:1000,3);
sfrcdat = sfrcst(901:1000,3);

// Regenerate the continuous signals.

from = 1:3;
to = [ LM(1:3,3) , LM(2:4,3) ]';

rmeas = REGENERATE(frcdat,Mfrcdat,sfrcdat,from,to);
```

Figure 7 Qualitative simulation of hydraulic subsystem.

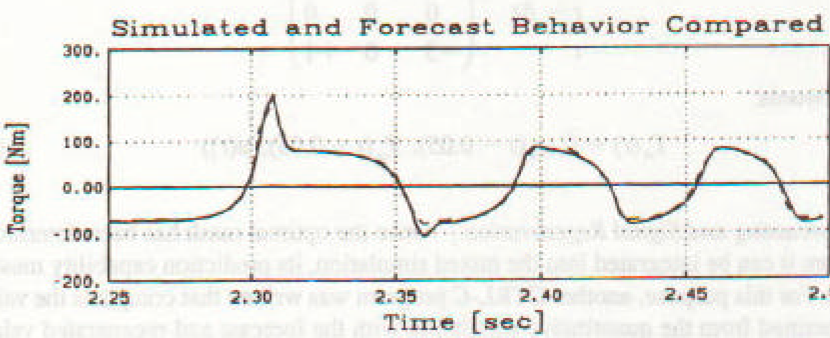

Figure 8 Simulated and forecast torque trajectories compared.

mal mask contains sufficient information about the behavior of the hydraulic subsystem to be used as a valid replacement of the true quantitative differential equation model, although the chosen recoding scheme was extremely crude using three levels for each variable only. Notice that the fuzzy inductive reasoning model was constructed solely on the basis of measurement data. No insight into the functioning of the hydraulic subsystem was required other than the knowledge that the torque, T_m, dynamically depends on the control signal, u, and the angular velocity, ω_m.

4.2. *Mixed Modeling and Simulation*

Once the prediction capability has been demonstrated, the fuzzy inductive reasoning model can be used to replace the former differential equation model of the hydraulic subsystem in a mixed simulation, where the electrical and mechanical subsystems are still modeled using differential equations, whereas the hydraulic subsystem is modeled using a fuzzy optimal mask. The mixed model is shown on Figure 9. The quantitative control signal, u, is converted to a qualitative triple, u^*, using fuzzy recoding (subsection 3.1). Also the quantitative angular velocity, ω_m, of the hydraulic motor is converted to a qualitative triple, ω_m^*. From these two qualitative signals, a qualitative triple of the torque of the hydraulic motor, T_m^*, is computed by means of fuzzy forecasting (subsection 3.2). This qualitative signal is then converted back to a quantitative signal, T_m, using fuzzy signal regeneration.

Forecasting was restricted to the last 100 sampling intervals, i.e., to the time span from 2.25 to 2.5 seconds. Figure 10 compares the angular position, θ_m, of the hydraulic motor from the purely quantitative simulation (solid line) with that of the mixed quantitative and

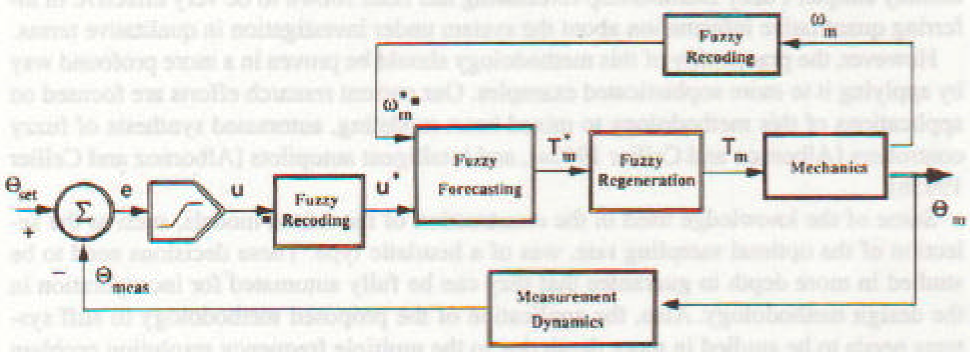

Figure 9 Mixed model of the hydraulic system.

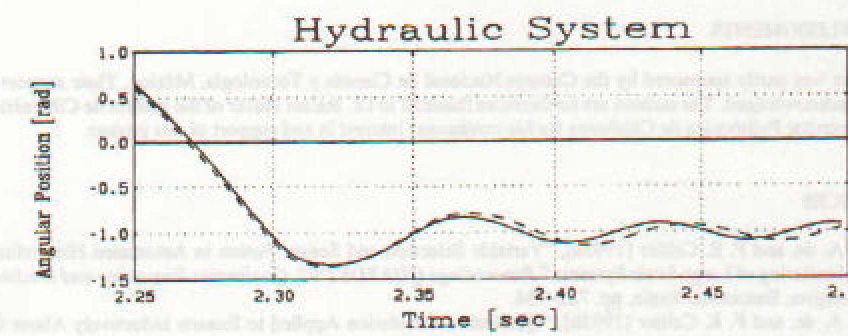

Figure 10 Comparison of quantitative and mixed simulations.

qualitative simulation (dashed line). As was to be expected, the mixed model behaves like a sampled-data control system. The mixed simulation exhibits an oscillation amplitude that is slightly larger and an oscillation frequency that is slightly smaller than those shown by the purely quantitative simulation. Surprisingly, the damping characteristics of the mixed model are slightly better than those of the purely quantitative model.

5. CONCLUSIONS

The example demonstrates the validity of the chosen approach. Mixed simulations are similar in effect to sampled-data system simulations. *Fuzzy recoding* takes the place of analog-to-digital converters, and *fuzzy signal regeneration* takes the place of digital-to-analog converters. However, this is where the similarity ends. Sampled-data systems operate on a fairly accurate representation of the digital signals. Typical converters are 12-bit converters, corresponding to discretized signals with 4096 discrete levels. In contrast, the fuzzy inductive reasoning model employed in the above example recoded all three variables into qualitative variables with the three levels 'small,' 'medium,' and 'large.' The quantitative information is retained in the fuzzy membership functions that accompany the qualitative signals. Due to the small number of discrete levels, the resulting finite state machine is extremely simple. Fuzzy membership forecasting has been shown to be very effective in inferring quantitative information about the system under investigation in qualitative terms.

However, the practicality of this methodology should be proven in a more profound way by applying it to more sophisticated examples. Our current research efforts are focused on applications of this methodology to mixed heart modeling, automated synthesis of fuzzy controllers [Albornoz and Cellier 1993a], and intelligent autopilots [Albornoz and Cellier 1993b].

Some of the knowledge used in the construction of the mixed models, such as the selection of the optimal sampling rate, was of a heuristic type. These decisions need to be studied in more depth to guarantee that they can be fully automated for incorporation in the design methodology. Also, the application of the proposed methodology to stiff systems needs to be studied in more depth due to the multiple frequency resolution problem inherent in such systems. It is not practical to simply request the mask to be made deeper and deeper.

ACKNOWLEDGMENTS

This project was partly sponsored by the Consejo Nacional de Ciencia y Tecnología, México. Their support is gratefully acknowledged. The authors are furthermore thankful to Dr. Rafael Huber of the Institut de Cibernètica of the Universitat Politècnica de Catalunya for his continuous interest in and support of this project.

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