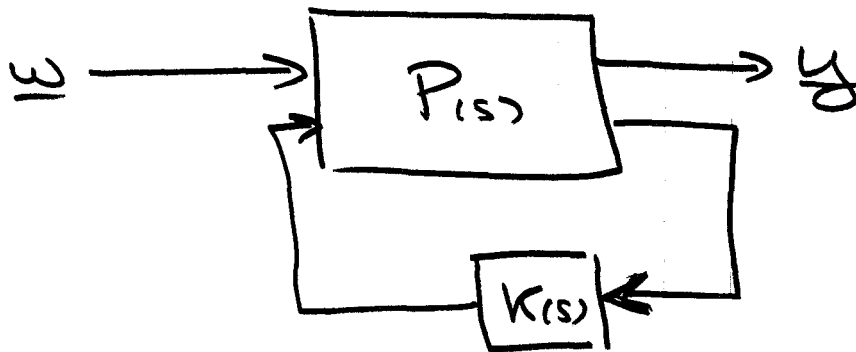


This is the robustness problem.

Linear Fractional Transforms:

Given :

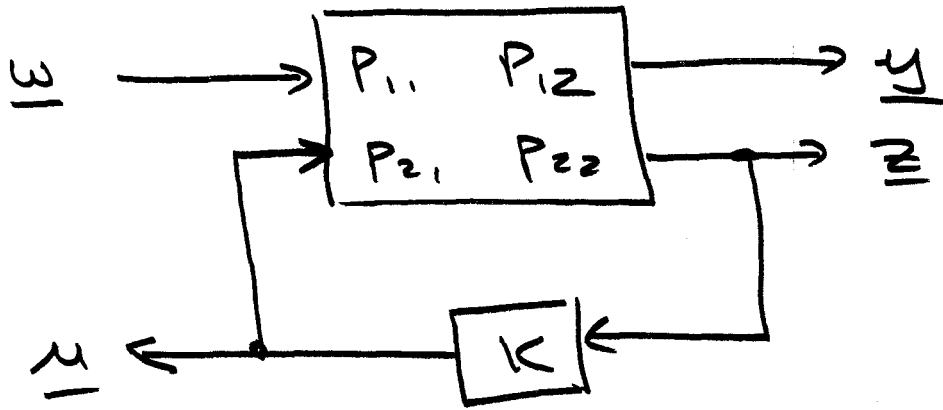


What is the transfer function from $w \rightarrow y$?

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

$P_{ij}(s)$ can still be multivariable !





$$\begin{cases} Y_1 = P_{11} \cdot U + P_{12} \cdot Y_2 \\ Y_2 = P_{21} \cdot U + P_{22} \cdot Y_2 \\ Y_2 = K \cdot Y_2 \end{cases}$$

$$\Rightarrow Y_2 = K \cdot Y_2 = K P_{21} U + K P_{22} Y_2$$

$$\Rightarrow [I - K P_{22}] Y_2 = K P_{21} U$$

$$\Rightarrow Y_2 = [I - K P_{22}]^{-1} \cdot K P_{21} U$$

$$\Rightarrow Y_1 = \left[P_{11} + P_{12} [I - K P_{22}]^{-1} K P_{21} \right] U$$

$$\Rightarrow \frac{Y_1}{U} (P, K) \equiv P_{11} + P_{12} [I - K P_{22}]^{-1} K P_{21}$$

So:

$$\underline{z} = P_{21} \underline{w} + P_{22} K \cdot \underline{z}$$

$$\Rightarrow [I - P_{22} K] \underline{z} = P_{21} \underline{w}$$

$$\Rightarrow \underline{z} = [I - P_{22} K]^{-1} P_{21} \underline{w}$$

$$\Rightarrow \underline{y} = K \underline{z} = K [I - P_{22} K]^{-1} P_{21} \underline{w}$$

$$\Rightarrow \underline{y} = [P_{11} + P_{12} K [I - P_{22} K]^{-1} P_{21}] \underline{w}$$

$$\Rightarrow \underline{F}_\ell(P, K) = P_{11} + P_{12} K [I - P_{22} K]^{-1} P_{21}$$

$\underline{F}_\ell(P, K)$ is the lower linear fractional transform of P fed back by K .

13-782 500 SHEETS, FILLER, 9 SQUARE
42-381 100 SHEETS, FILLER, 9 SQUARE
42-382 100 SHEETS, FILLER, 9 SQUARE
42-383 100 SHEETS, FILLER, 9 SQUARE
42-384 100 SHEETS, FILLER, 9 SQUARE
42-385 100 SHEETS, FILLER, 9 SQUARE
42-386 100 SHEETS, FILLER, 9 SQUARE
42-387 100 SHEETS, FILLER, 9 SQUARE
42-388 100 SHEETS, FILLER, 9 SQUARE
42-389 100 SHEETS, FILLER, 9 SQUARE
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42-391 100 SHEETS, FILLER, 9 SQUARE
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42-393 100 SHEETS, FILLER, 9 SQUARE
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42-396 100 SHEETS, FILLER, 9 SQUARE
42-397 100 SHEETS, FILLER, 9 SQUARE
42-398 100 SHEETS, FILLER, 9 SQUARE
42-399 100 SHEETS, FILLER, 9 SQUARE
42-400 100 SHEETS, FILLER, 9 SQUARE
Printed in U.S.A.



In the time domain:

$$P(s) = \left[\begin{array}{c|cc} A_p & B_{p1} & B_{p2} \\ \hline C_{p1} & D_{p11} & D_{p12} \\ C_{p2} & D_{p21} & D_{p22} \end{array} \right]$$

$$K(s) = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array} \right]$$

$$\dot{X}_p = D_p X_p + B_{p1} U + B_{p2} \dot{U}$$

$$I \dot{U} = C_{p1} X_p + D_{p11} U + D_{p12} \dot{U}$$

$$I \dot{U} = C_{p2} X_p + D_{p21} U + D_{p22} \dot{U}$$

$$\dot{X}_k = D_k X_k + B_k \dot{U}$$

$$Y = C_k X_k + D_k \dot{U}$$

$$Y = C_k X_k + D_k C_{p2} X_p + D_k D_{p21} U + D_k D_{p22} \dot{U}$$

$$\Rightarrow \underbrace{(I - D_k D_{p22})}_{DD} Y = C_k X_k + D_k C_{p2} X_p + D_k D_{p21} U$$

$$\Rightarrow Y = DD^{-1} D_k C_{p2} X_p + DD^{-1} C_k X_k + DD^{-1} D_k D_{p21} U$$

13-762 500 SHEETS, FILLER, 5 SQUARE
 42-381 50 SHEETS, EYE-LEASER, 5 SQUARE
 42-382 100 SHEETS, EYE-LEASER, 5 SQUARE
 42-383 100 SHEETS, EYE-LEASER, 5 SQUARE
 42-384 100 RECYCLED WHITE, 5 SQUARE
 42-385 200 RECYCLED WHITE, 5 SQUARE
 Made in U.S.A.

 National Brand

$$\Rightarrow \dot{X}_P = [A_P + B_{P2} \cdot D D^{-1} \cdot D_K C_{P2}] X_P + [B_{P2} \cdot D D^{-1} \cdot C_K] X_K + [B_{P1} + B_{P2} \cdot D D^{-1} \cdot D_{1c} D_{P21}] \underline{W}$$

$$\dot{X}_K = D_K C_{P2} X_P + A_K X_K + B_K D_{P21} \underline{W} + B_{1c} D_{P22} \underline{U}$$

$$\Rightarrow \dot{X}_K = [D_K C_{P2} + B_K D_{P22} \cdot D D^{-1} \cdot D_K C_{P2}] X_P + [A_K + B_K D_{P22} \cdot D D^{-1} \cdot C_K] X_K + [B_{1c} D_{P21} + B_{1c} D_{P22} \cdot D D^{-1} \cdot D_K D_{P21}] \underline{W}$$

$$\underline{Y} = [C_{P1} + D_{P12} \cdot D D^{-1} \cdot D_K C_{P2}] X_P + [D_{P12} \cdot D D^{-1} \cdot C_K] X_K + [D_{P11} + D_{P12} \cdot D D^{-1} \cdot D_K D_{P21}] \underline{W}$$

$$\Rightarrow F_r(P,K) = \frac{\begin{bmatrix} (A_P + B_{P2} D D^{-1} D_K C_{P2}) & (B_{P2} D D^{-1} C_K) \\ (D_K C_{P2} + B_K D_{P22} D D^{-1} D_K C_{P2}) & (A_K + B_K D_{P22} D D^{-1} C_K) \end{bmatrix}}{\begin{bmatrix} (C_{P1} + D_{P12} D D^{-1} D_K C_{P2}) & (D_{P12} D D^{-1} C_K) \end{bmatrix}}$$

$$\frac{\begin{bmatrix} (B_{P1} + B_{P2} D D^{-1} D_K D_{P21}) \\ (B_{1c} D_{P21} + B_{1c} D_{P22} D D^{-1} D_{1c} D_{P21}) \end{bmatrix}}{(D_{P11} + D_{P12} D D^{-1} D_K D_{P21})}$$

$$P_p(s) \cdot \underline{x}_p(t) = Q_{p_1}(s) \cdot \underline{w}(t) + Q_{p_2}(s) \cdot \underline{u}(t)$$

$$\underline{y}(t) = R_{p_1}(s) \cdot \underline{x}_p(t) + W_{p_{11}}(s) \cdot \underline{w}(t) + W_{p_{12}}(s) \cdot \underline{u}(t)$$

$$\underline{z}(t) = R_{p_2}(s) \cdot \underline{x}_p(t) + W_{p_{21}}(s) \cdot \underline{w}(t) + W_{p_{22}}(s) \cdot \underline{u}(t)$$

$$P_k(s) \cdot \underline{x}_k(t) = Q_k(s) \cdot \underline{z}(t)$$

$$\underline{u}(t) = R_k(s) \cdot \underline{x}_k(t) + W_k(s) \cdot \underline{z}(t)$$

13-7502 500 SHEETS, FULLER 5 SQUARE
42-351 40 SHEETS, EYEGLASS 5 SQUARE
42-352 20 SHEETS, EYEGLASS 5 SQUARE
42-353 10 SHEETS, EYEGLASS 5 SQUARE
42-354 5 SHEETS, EYEGLASS 5 SQUARE
42-355 100 RECYCLED WHITE 5 SQUARE
42-356 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.



$$P_p(s) \cdot \underline{x}_p(t) - Q_{p_2}(s) \cdot \underline{u}(t) = Q_{p_1}(s) \cdot \underline{w}(t)$$

$$P_k(s) \cdot \underline{x}_k(t) - Q_k(s) \cdot \underline{z}(t) = \emptyset$$

$$-R_k(s) \cdot \underline{x}_k(t) - W_k(s) \cdot \underline{z}(t) + \underline{u}(t) = \emptyset$$

$$-R_{p_2}(s) \cdot \underline{x}_p(t) + \underline{z}(t) - W_{p_{22}}(s) \cdot \underline{u}(t) = W_{p_{21}}(s) \cdot \underline{w}(t)$$

$$\underline{y}(t) = R_{p_1}(s) \cdot \underline{x}_p(t) + W_{p_{12}}(s) \cdot \underline{u}(t) + W_{p_{11}}(s) \cdot \underline{w}(t)$$

Let: $\underline{v}(t) = \begin{bmatrix} \underline{x}_p(t) \\ \underline{x}_k(t) \\ -\underline{z}(t) \\ -\underline{u}(t) \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} P_p(s) & \emptyset & \emptyset & Q_{p_2}(s) \\ \emptyset & P_k(s) & Q_k(s) & \emptyset \\ \emptyset & -R_k(s) & W_k(s) & -I \\ -R_{p_2}(s) & \emptyset & -I & W_{p_{22}}(s) \end{bmatrix} \underline{w}(t) = \begin{bmatrix} Q_{p_1}(s) \\ \emptyset \\ \emptyset \\ W_{p_{21}}(s) \end{bmatrix} \underline{w}(t)$$

$$\underline{y}(t) = \begin{bmatrix} R_{p_1}(s) & \emptyset & \emptyset & -W_{p_{12}}(s) \end{bmatrix} \underline{w}(t) + \begin{bmatrix} W_{p_{11}}(s) \end{bmatrix} \underline{w}(t)$$

is a polynomial matrix system representation of $\mathcal{F}_2(P, K)$.

$$\Rightarrow S(s) = \begin{bmatrix} \begin{bmatrix} P_p(s) & \emptyset & \emptyset & Q_{p_2}(s) \\ \emptyset & P_k(s) & Q_k(s) & \emptyset \\ \emptyset & -R_k(s) & W_k(s) & -I \\ -R_{p_2}(s) & \emptyset & -I & W_{p_{22}}(s) \end{bmatrix} & \begin{bmatrix} Q_{p_1}(s) \\ \emptyset \\ \emptyset \\ W_{p_{21}}(s) \end{bmatrix} \\ \begin{bmatrix} R_{p_1}(s) & \emptyset & \emptyset & -W_{p_{12}}(s) \end{bmatrix} & \begin{bmatrix} W_{p_{11}}(s) \end{bmatrix} \end{bmatrix}$$

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```
function [T] = lft1(G,K)
```

```
%  
% This function operates on POLPAC system descriptions stored in modes 0,  
% 4..6, and 7..9. T is the lower linear fractional transformation (LFT)  
% of G and K.
```

```
% Mixed mode operations are handled with a preference of trajectories  
% over coefficients over roots.
```

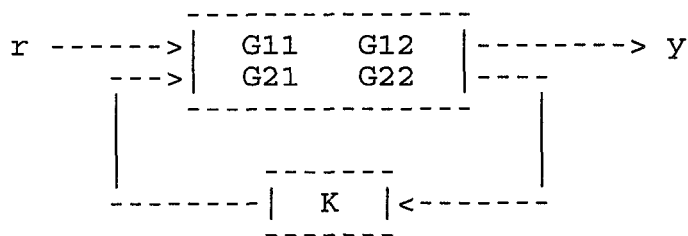
```
% Input Parameters:  
% -----
```

```
% G      := System in forward path  
% K      := System in feedback path
```

```
% Output Parameter:  
% -----
```

```
% T      := Transmission system ( r -> y )
```

```
% Explanation:  
% -----
```



```
% push('LFTL')  
% global debg
```

```
% Check for necessary conversions first
```

```
% [G,K] = mixmod(G,K);
```

```
% Start by unpacking the structural information
```

```
% [modG,lcolG,dtypG,stypG] = unpack(G(1,1));  
% [modK,lcolK,dtypK,stypK] = unpack(K(1,1));  
logcol = max([lcolG,lcolK]);  
[pG,mG] = logdim(G);  
[pK,mK] = logdim(K);  
domG = domain(G);
```

```
% Check for consistency
```

```
% if debg == 3,  
%   if modG > 0,  
%     if modG < 4,  
%       disp('LFTU: Error - Operates on system modes only'),  
%       abort,  
%     end,  
%   end,  
%   if stypK ~= stypG,  
%     disp('LFTL: Error - Incompatible system types'),  
%     abort,
```

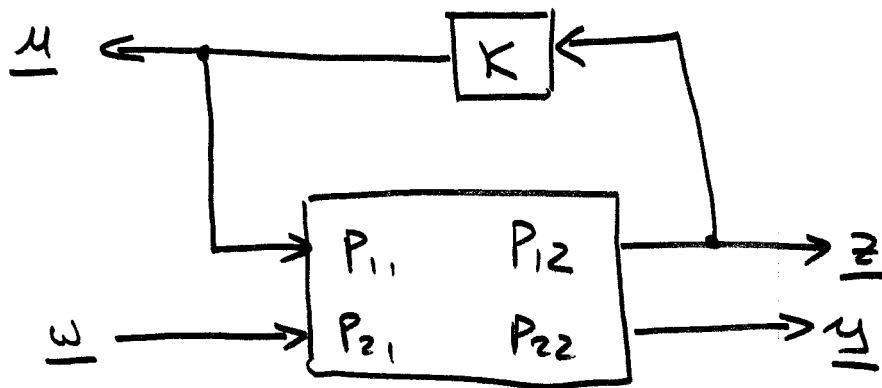


```
end,
end
if debg >= 2,
    if mG <= pK,
        disp('LFTL: Error - Incompatible number of rows/columns'),
        abort,
    end,
    if pG <= mK,
        disp('LFTL: Error - Incompatible number of rows/columns'),
        abort,
    end,
end
end
%
% Branch depending on the mode of the system.
%
if modG == 0,
    %
    % Is a state-space description
    %
    [Ag,Bg,Cg,Dg] = oldsysst(G);
    [Ak,Bk,Ck,Dk] = oldsysst(K);
    %
    % Extract the four subsystems.
    %
    Bg1 = Bg(:,1:mG-pK);
    Bg2 = Bg(:,mG-pK+1:mG);
    Cg1 = Cg(1:pG-mK,:);
    Cg2 = Cg(pG-mK+1:pG,:);
    Dg11 = Dg(1:pG-mK,1:mG-pK);
    Dg12 = Dg(1:pG-mK,mG-pK+1:mG);
    Dg21 = Dg(pG-mK+1:pG,1:mG-pK);
    Dg22 = Dg(pG-mK+1:pG,mG-pK+1:mG);
    DkDg22 = Dk*Dg22;
    DD = eye(size(DkDg22)) - DkDg22;
    %
    % Check whether the problem is well-posed
    %
    if norm(DD) < 1000*eps,
        disp('LFTL: Error - System is ill-posed'),
        abort,
        return,
    end,
    %
    % Now compute the feedback structure
    %
    DDinv = inv(DD);
    Bg2DDi = Bg2*DDinv;
    BkDg22DDi = Bk*Dg22*DDinv;
    DkCg2 = Dk*Cg2;
    DkDg21 = Dk*Dg21;
    Bcl1 = Bg1 + Bg2DDi*DkDg21;
    Bcl2 = Bk*Dg21 + BkDg22DDi*DkDg21;
    Bcl = [ Bcl1 ; Bcl2 ];
    Acl11 = Ag + Bg2DDi*DkCg2;
    Acl12 = Bg2DDi*Ck;
    Acl21 = Bk*Cg2 + BkDg22DDi*DkCg2;
    Acl22 = Ak + BkDg22DDi*Ck;
    Acl = [ Acl11 , Acl12 ; Acl21 , Acl22 ];
    Dg12DDi = Dg12*DDinv;
    Dcl = Dg11 + Dg12DDi*DkDg21;
```

```
Ccl1 = Cg1 + Dg12DDi*DkCg2;
Ccl2 = Dg12DDi*Ck;
Ccl = [ Ccl1 , Ccl2 ];
T = newsyst(Acl,Bcl,Ccl,Dcl,stypG);
%
% Get rid of uncontrollable/unobservable modes
%
T = minreals(T);
pull,
return,
end
%
if modG == 4 | modG == 5 | modG == 6,
%
% Is a transfer function matrix
% Extract the four submatrices.
%
G11 = gget(G,1:pG-mK,1:mG-pK);
G12 = gget(G,1:pG-mK,mG-pK+1:mG);
G21 = gget(G,pG-mK+1:pG,1:mG-pK);
G22 = gget(G,pG-mK+1:pG,mG-pK+1:mG);
%
% Now compute: T = G11 + G12*K*(I - G22*K)\G21
%
Aux = multg(G22,K);
I = eyep(mK,mK,modG,logcol,stypG,domG);
Aux = sub(I,Aux);
%
% Check whether system is well-posed
%
[G2,P2] = strictprop(Aux);
ord = order(P2);
if ord == 0,
    gn = gain(P2);
    if gn == 0,
        disp('LFTL: Error - System is ill-posed'),
        abort,
        return,
    end,
end,
Aux = invg(Aux);
Aux = multg(Aux,G21);
Aux = multg(K,Aux);
Aux = multg(G12,Aux);
T = addg(G11,Aux);
%
% Get rid of uncontrollable/unobservable modes
%
T = reduceeg(T);
pull,
return,
end
%
if modG == 7 | modG == 8 | modG == 9,
%
% Is a polynomial matrix system description
%
[Pg,Qg,Rg,Wg] = pms2plm(G);
[Pk,Qk,Rk,Wk] = pms2plm(K);
%
```

```
% Extract the four subsystems
%
[nQ,mQ] = logdim(Qg);
[nR,mR] = logdim(Rg);
Qg1 = gget(Qg,1:nQ,1:mG-pK);
Qg2 = gget(Qg,1:nQ,mG-pK+1:mG);
Rg1 = gget(Rg,1:pG-mK,1:mR);
Rg2 = gget(Rg,pG-mK+1:pG,1:mR);
Wg11 = gget(Wg,1:pG-mK,1:mG-pK);
Wg12 = gget(Wg,1:pG-mK,mG-pK+1:mG);
Wg21 = gget(Wg,pG-mK+1:pG,1:mG-pK);
Wg22 = gget(Wg,pG-mK+1:pG,mG-pK+1:mG);
%
% Plug closed loop polynomial matrices together
%
[qG,qG] = logdim(Pg);
[qK,qK] = logdim(Pk);
qT = qG + qK + mK + pK;
pT = pG - mK;
mT = mG - pK;
Pcl = zerop(qT,qT,modG-6,logcol,stypG,domG);
Im = eyep(mK,mK,modG-6,logcol,stypG,domG);
Ip = eyep(pK,pK,modG-6,logcol,stypG,domG);
Pcl = put(Pcl,Pg,1:qG,1:qG);
Pcl = put(Pcl,Qg2,1:qG,qT-pK+1:qT);
Pcl = put(Pcl,Pk,qG+1:qG+qK,qG+1:qG+qK);
Pcl = put(Pcl,Qk,qG+1:qG+qK,qG+qK+1:qT-pK);
Pcl = put(Pcl,minus(Rk),qG+qK+1:qT-mK,qG+1:qG+qK);
Pcl = put(Pcl,Wk,qG+qK+1:qT-mK,qG+qK+1:qT-pK);
Pcl = put(Pcl,minus(Ip),qG+qK+1:qT-mK,qT-pK+1:qT);
Pcl = put(Pcl,minus(Rg2),qT-mK+1:qT,1:qG);
Pcl = put(Pcl,minus(Im),qT-mK+1:qT,qG+qK+1:qT-pK);
Pcl = put(Pcl,Wg22,qT-mK+1:qT,qT-pK+1:qT);
Qcl = zerop(qT,mT,modG-6,logcol,stypG,domG);
Qcl = put(Qcl,Qg1,1:qG,1:mT);
Qcl = put(Qcl,Wg21,qT-mK+1:qT,1:mT);
Rcl = zerop(pT,qT,modG-6,logcol,stypG,domG);
Rcl = put(Rcl,Rg1,1:pT,1:qG);
Rcl = put(Rcl,minus(Wg12),1:pT,qT-pK+1:qT);
Wcl = Wg11;
%
% Make a new polynomial matrix system
%
T = plm2pms(Pcl,Qcl,Rcl,Wcl);
%
% Get rid of uncontrollable/unobservable modes
%
T = minrealps(T);
pull,
return,
end
%
return
```

Analogous:



$$\begin{cases} Z = P_{11} \cdot U + P_{12} \cdot W \\ Y = P_{21} \cdot U + P_{22} \cdot W \\ U = K \cdot Z \end{cases}$$

$$\Rightarrow U = K \cdot Z = K \cdot P_{11} \cdot U + K \cdot P_{12} \cdot W$$

$$\Rightarrow [I - K \cdot P_{11}] \cdot U = K \cdot P_{12} \cdot W$$

$$\Rightarrow U = [I - K \cdot P_{11}]^{-1} \cdot K \cdot P_{12} \cdot W$$

$$\Rightarrow Y = [P_{22} + P_{21} \cdot [I - K \cdot P_{11}]^{-1} \cdot K \cdot P_{12}] \cdot W$$

$$\Rightarrow \underline{F}_u(P, K) = P_{22} + P_{21} \cdot [I - K \cdot P_{11}]^{-1} \cdot K \cdot P_{12}$$

$$\begin{aligned} \underline{z} &= P_{11} \cdot \underline{y} + P_{12} \cdot \underline{w} \\ &= P_{11} \cdot K \cdot \underline{z} + P_{12} \cdot \underline{w} \end{aligned}$$

$$\Rightarrow [I - P_{11} \cdot K] \underline{z} = P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{z} = [I - P_{11} \cdot K]^{-1} \cdot P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{y} = K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{y} = [P_{22} + P_{21} \cdot K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12}] \underline{w}$$

$$\Rightarrow \underline{F}_u(P, K) = P_{22} + P_{21} \cdot K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12}$$

$\underline{F}_u(P, K)$ is the upper linear fractional transform of P fed back by K .



In the time domain:

$$P(s) = \left[\begin{array}{c|cc} A_P & B_{P1} & B_{P2} \\ \hline C_{P1} & D_{P11} & D_{P12} \\ C_{P2} & D_{P21} & D_{P22} \end{array} \right]$$

$$K(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array} \right]$$

$$\left. \begin{array}{l} \dot{X}_P = A_P X_P + B_{P1} U + B_{P2} W \\ W = C_{P1} X_P + D_{P11} U + D_{P12} W \\ U = C_{P2} X_P + D_{P21} U + D_{P22} W \\ \dot{X}_K = A_K X_K + B_K W \\ U = C_K X_K + D_K W \end{array} \right|$$

$$U = C_K X_K + D_K C_{P1} X_P + D_K D_{P11} U + D_K D_{P12} W$$

$$\Rightarrow \underbrace{(I - D_K D_{P11})}_{DD} U = D_K C_{P1} X_P + C_K X_K + D_K D_{P12} W$$

$$\Rightarrow \underline{U} = DD^{-1} \cdot D_K C_{P1} X_P + DD^{-1} \cdot C_K X_K + DD^{-1} \cdot D_K D_{P12} W$$

$$\dot{\underline{x}}_p = [A_p + B_{p1} \cdot D D^{-1} \cdot D_k C_{p1}] \underline{x}_p + [B_{p1} \cdot D D^{-1} \cdot C_k] \underline{x}_k + [B_{p2} + B_{p1} \cdot D D^{-1} \cdot D_k D_{p12}] \underline{w}$$

$$\dot{\underline{x}}_k = [B_k C_{p1}] \underline{x}_p + A_k \underline{x}_k + [B_k D_{p12}] \underline{w} + [B_k D_{p11}] \underline{u}$$

$$\begin{aligned} \Rightarrow \dot{\underline{x}}_k &= [B_k C_{p1} + B_k D_{p11} \cdot D D^{-1} \cdot D_k C_{p1}] \underline{x}_p \\ &+ [A_k + B_k D_{p11} \cdot D D^{-1} \cdot C_k] \underline{x}_k \\ &+ [B_k D_{p12} + B_k D_{p11} \cdot D D^{-1} \cdot D_k D_{p12}] \underline{w} \end{aligned}$$

$$\underline{y} = [C_{p2} + D_{p21} \cdot D D^{-1} \cdot D_k C_{p1}] \underline{x}_p + [D_{p21} \cdot D D^{-1} \cdot C_k] \underline{x}_k + [D_{p22} + D_{p21} \cdot D D^{-1} \cdot D_k D_{p12}] \underline{w}$$

$$\Rightarrow F_u(p, k) = \frac{\begin{bmatrix} (A_p + B_{p1} D D^{-1} D_k C_{p1}) & (B_{p1} D D^{-1} C_k) \\ (B_k C_{p1} + B_k D_{p11} D D^{-1} D_k C_{p1}) & (A_k + B_k D_{p11} D D^{-1} C_k) \end{bmatrix}}{\begin{bmatrix} (C_{p2} + D_{p21} D D^{-1} D_k C_{p1}) & (D_{p21} D D^{-1} C_k) \end{bmatrix}}$$

$$\frac{\begin{bmatrix} (B_{p2} + B_{p1} D D^{-1} D_k D_{p12}) \\ (B_k D_{p12} + B_k D_{p11} D D^{-1} D_k D_{p12}) \end{bmatrix}}{(D_{p22} + D_{p21} D D^{-1} D_k D_{p12})}$$

$$P_p(s) \cdot \underline{x}_p(t) = Q_{p_1}(s) \cdot \underline{u}(t) + Q_{p_2}(s) \cdot \underline{w}(t)$$

$$\underline{z}(t) = R_{p_1}(s) \cdot \underline{x}_p(t) + W_{p_{11}}(s) \cdot \underline{u}(t) + W_{p_{12}}(s) \cdot \underline{w}(t)$$

$$\underline{y}(t) = R_{p_2}(s) \cdot \underline{x}_p(t) + W_{p_{21}}(s) \cdot \underline{u}(t) + W_{p_{22}}(s) \cdot \underline{w}(t)$$

$$P_k(s) \cdot \underline{x}_k(t) = Q_k(s) \cdot \underline{z}(t)$$

$$\underline{u}(t) = R_k(s) \cdot \underline{x}_k(t) + W_k(s) \cdot \underline{z}(t)$$

$$P_p(s) \cdot \underline{x}_p(t) - Q_{p_1}(s) \cdot \underline{u}(t) = Q_{p_2}(s) \cdot \underline{w}(t)$$

$$P_k(s) \cdot \underline{x}_k(t) - Q_k(s) \cdot \underline{z}(t) = 0$$

$$-R_k(s) \cdot \underline{x}_k(t) - W_k(s) \cdot \underline{z}(t) + \underline{u}(t) = 0$$

$$-R_{p_1}(s) \cdot \underline{x}_p(t) + \underline{z}(t) - W_{p_{11}}(s) \cdot \underline{u}(t) = W_{p_{12}}(s) \cdot \underline{w}(t)$$

$$\underline{y}(t) = R_{p_2}(s) \cdot \underline{x}_p(t) + W_{p_{21}}(s) \cdot \underline{u}(t) + W_{p_{22}}(s) \cdot \underline{w}(t)$$

Let: $\underline{w} = \begin{bmatrix} \underline{x}_p(t) \\ \underline{x}_k(t) \\ -\underline{z}(t) \\ -\underline{u}(t) \end{bmatrix}$

⇒

$$\begin{bmatrix} P_p(s) & \emptyset & \emptyset & Q_{p_1}(s) \\ \emptyset & P_k(s) & Q_k(s) & \emptyset \\ \emptyset & -R_k(s) & W_k(s) & -I \\ -R_{p_1}(s) & \emptyset & -I & W_{p_{11}}(s) \end{bmatrix} \underline{u}(t) = \begin{bmatrix} Q_{p_2}(s) \\ \emptyset \\ \emptyset \\ W_{p_{12}}(s) \end{bmatrix} \underline{w}(t)$$

$$\underline{y}(t) = \begin{bmatrix} R_{p_2}(s) & \emptyset & \emptyset & -W_{p_{21}}(s) \end{bmatrix} \underline{u}(t) + \begin{bmatrix} W_{p_{22}}(s) \end{bmatrix} \underline{w}(t)$$

is a polynomial matrix system representation of $\mathcal{F}_u(P, K)$.