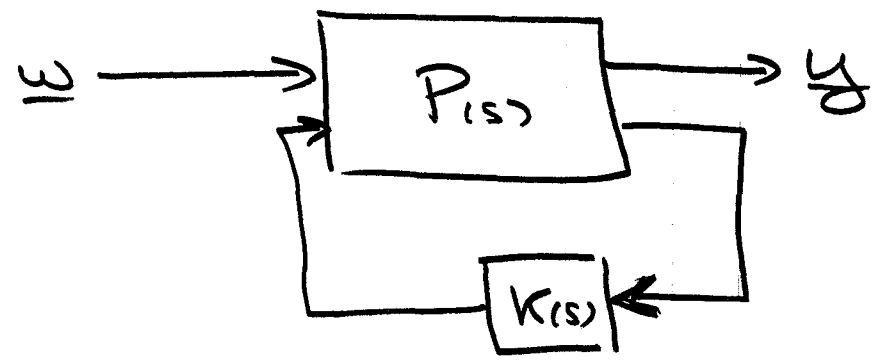
This is the <u>robustness</u> problem.

<u>Linear Fractional Transforms:</u>

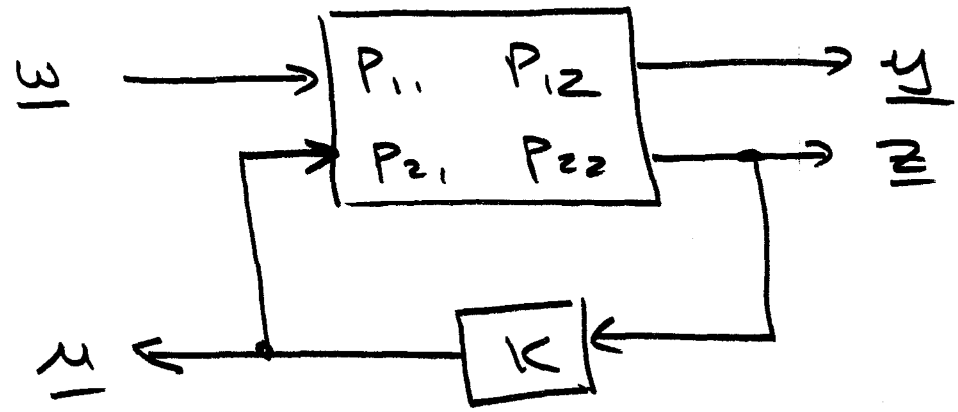
Given :



What is the transfer function from $\underline{w} \rightarrow \underline{y}$?

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

$P_{ij}(s)$ can still be multivariable !



$$\left|\begin{array}{l} \underline{y} = P_{11}\cdot\underline{w} + P_{12}\,\underline{u} \\ \underline{z} = P_{21}\cdot\underline{w} + P_{22}\cdot\underline{u} \\ \underline{u} = K\cdot\underline{z} \end{array}\right|$$

$$\Rightarrow \underline{u} = K\cdot\underline{z} = K P_{21}\,\underline{w} + K P_{22}\cdot\underline{u}$$

$$\Rightarrow \left[I - K P_{22}\right]\underline{u} = K P_{21}\,\underline{w}$$

$$\Rightarrow \underline{u} = \left[I - K P_{22}\right]^{-1}\cdot K P_{21}\,\underline{w}$$

$$\Rightarrow \underline{y} = \left[P_{11} + P_{12}\left[I - K P_{22}\right]^{-1} K P_{21}\right]\underline{w}$$

$$\Rightarrow \mathcal{F}_{\ell}(P,K) \equiv P_{11} + P_{12}\left[I - K\cdot P_{22}\right]^{-1} K P_{21}$$

So:

$$\underline{z} = P_{21} \cdot \underline{w} + P_{22} K \cdot \underline{z}$$

$$\Rightarrow [I - P_{22} K] \underline{z} = P_{21} \underline{w}$$

$$\Rightarrow \underline{z} = [I - P_{22} K]^{-1} P_{21} \underline{w}$$

$$\Rightarrow \underline{u} = K \underline{z} = K [I - P_{22} K]^{-1} P_{21} \underline{w}$$

$$\Rightarrow \underline{y} = \left[P_{11} + P_{12} K [I - P_{22} K]^{-1} P_{21} \right] \underline{w}$$

$$\Rightarrow \mathcal{F}_l(P, K) = P_{11} + P_{12} K [I - P_{22} K]^{-1} P_{21} \quad //$$

$\mathcal{F}_l(P,K)$ is the <u>lower linear fractional transform</u> of P fed back by K.

In the time domain:

$$P(s) = \left[\begin{array}{c|cc} A_p & B_{p1} & B_{p2} \\ \hline C_{p1} & D_{p11} & D_{p12} \\ C_{p2} & D_{p21} & D_{p22} \end{array}\right]$$

$$K(s) = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right]$$

$$\left|\begin{array}{l} \dot{\underline{x}}_p = A_p \underline{x}_p + B_{p1} \underline{w} + B_{p2} \underline{u} \\ \underline{y} = C_{p1} \underline{x}_p + D_{p11} \underline{w} + D_{p12} \underline{u} \\ \underline{z} = C_{p2} \underline{x}_p + D_{p21} \underline{w} + D_{p22} \underline{u} \\ \dot{\underline{x}}_k = A_k \underline{x}_k + B_k \underline{z} \\ \underline{u} = C_k \underline{x}_k + D_k \underline{z} \end{array}\right|$$

$$\underline{u} = C_k \underline{x}_k + D_k C_{p2} \underline{x}_p + D_k D_{p21} \underline{w} + D_k D_{p22} \underline{u}$$

$$\Rightarrow \underbrace{(I - D_k D_{p22})}_{DD} \underline{u} = C_k \underline{x}_k + D_k C_{p2} \underline{x}_p + D_k D_{p21} \underline{w}$$

$$\Rightarrow \underline{u} = DD^{-1} \cdot D_k C_{p2} \underline{x}_p + DD^{-1} \cdot C_k \underline{x}_k + DD^{-1} \cdot D_k D_{p21} \underline{w}$$

$$\Rightarrow \quad \underline{\dot{x}}_p = \left[A_p + B_{p_2} \cdot DD^{-1} \cdot D_k C_{p_2}\right] \underline{x}_p + \left[B_{p_2} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k$$
$$+ \left[B_{p_1} + B_{p_2} \cdot DD^{-1} \cdot D_k D_{p_{21}}\right] \underline{\omega}$$

$$\underline{\dot{x}}_k = B_k C_{p_2} \underline{x}_p + A_k \underline{x}_k + B_k D_{p_{21}} \underline{\omega} + B_k D_{p_{22}} \underline{u}$$

$$\Rightarrow \underline{\dot{x}}_k = \left[B_k C_{p_2} + B_k D_{p_{22}} \cdot DD^{-1} \cdot D_k C_{p_2}\right] \underline{x}_p$$
$$+ \left[A_k + B_k D_{p_{22}} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k$$
$$+ \left[B_k D_{p_{21}} + B_k D_{p_{22}} \cdot DD^{-1} \cdot D_k D_{p_{21}}\right] \underline{\omega}$$

$$\underline{y} = \left[C_{p_1} + D_{p_{12}} \cdot DD^{-1} \cdot D_k C_{p_2}\right] \underline{x}_p + \left[D_{p_{12}} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k$$
$$+ \left[D_{p_{11}} + D_{p_{12}} \cdot DD^{-1} \cdot D_k D_{p_{21}}\right] \underline{\omega}$$

$$\Rightarrow \mathcal{F}_\ell(P,K) = \left[\begin{array}{cc|c} (A_p + B_{p_2} DD^{-1} D_k C_{p_2}) & (B_{p_2} DD^{-1} C_k) & (B_{p_1} + B_{p_2} DD^{-1} D_k D_{p_{21}}) \\ (D_k C_{p_2} + B_k D_{p_{22}} DD^{-1} D_k C_{p_2}) & (A_k + B_k D_{p_{22}} DD^{-1} C_k) & (B_k D_{p_{21}} + B_k D_{p_{22}} DD^{-1} D_k D_{p_{21}}) \\ \hline (C_{p_1} + D_{p_{12}} DD^{-1} D_k C_{p_2}) & (D_{p_{12}} DD^{-1} C_k) & (D_{p_{11}} + D_{p_{12}} DD^{-1} D_k D_{p_{21}}) \end{array}\right]$$

$$\begin{cases} P_p(s)\cdot \underline{x}_p(t) = Q_{p_1}(s)\cdot \underline{w}(t) + Q_{p_2}(s)\cdot \underline{u}(t) \\ \underline{y}(t) = R_{p_1}(s)\cdot \underline{x}_p(t) + W_{p_{11}}(s)\cdot \underline{w}(t) + W_{p_{12}}(s)\cdot \underline{u}(t) \\ \underline{z}(t) = R_{p_2}(s)\cdot \underline{x}_p(t) + W_{p_{21}}(s)\cdot \underline{w}(t) + W_{p_{22}}(s)\cdot \underline{u}(t) \\ P_k(s)\cdot \underline{x}_k(t) = Q_k(s)\cdot \underline{z}(t) \\ \underline{u}(t) = R_k(s)\cdot \underline{x}_k(t) + W_k(s)\cdot \underline{z}(t) \end{cases}$$

$$\Rightarrow \begin{cases} P_p(s)\cdot \underline{x}_p(t) - Q_{p_2}(s)\cdot \underline{u}(t) = Q_{p_1}(s)\cdot \underline{w}(t) \\ P_k(s)\cdot \underline{x}_k(t) - Q_k(s)\cdot \underline{z}(t) = \emptyset \\ -R_k(s)\cdot \underline{x}_k(t) - W_k(s)\cdot \underline{z}(t) + \underline{u}(t) = \emptyset \\ -R_{p_2}(s)\cdot \underline{x}_p(t) + \underline{z}(t) - W_{p_{22}}(s)\cdot \underline{u}(t) = W_{p_{21}}(s)\cdot \underline{w}(t) \\ \underline{y}(t) = R_{p_1}(s)\cdot \underline{x}_p(t) + W_{p_{12}}(s)\cdot \underline{u}(t) + W_{p_{11}}(s)\cdot \underline{w}(t) \end{cases}$$

Let: $\underline{\xi}(t) = \begin{bmatrix} \underline{x}_p(t) \\ \underline{x}_k(t) \\ -\underline{z}(t) \\ -\underline{u}(t) \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} P_p(s) & \Phi & \Phi & Q_{p_2}(s) \\ \Phi & P_k(s) & Q_k(s) & \Phi \\ \Phi & -R_k(s) & W_k(s) & -I \\ -R_{p_2}(s) & \Phi & -I & W_{p_{22}}(s) \end{bmatrix} \underline{\xi}(t) = \begin{bmatrix} Q_{p_1}(s) \\ \Phi \\ \Phi \\ W_{p_{21}}(s) \end{bmatrix} \underline{w}(t)$$

$$\underline{y}(t) = \begin{bmatrix} R_{p_1}(s) & \Phi & \Phi & -W_{p_{12}}(s) \end{bmatrix} \underline{\xi}(t) + \begin{bmatrix} W_{p_{11}}(s) \end{bmatrix} \underline{w}(t)$$

is a polynomial matrix system representation of $\mathcal{F}_\ell(P,K)$.

$$\Rightarrow S(s) = \left[\begin{array}{c|c} \begin{bmatrix} P_p(s) & \Phi & \Phi & Q_{p_2}(s) \\ \Phi & P_k(s) & Q_k(s) & \Phi \\ \Phi & -R_k(s) & W_k(s) & -I \\ -R_{p_2}(s) & \Phi & -I & W_{p_{22}}(s) \end{bmatrix} & \begin{bmatrix} Q_{p_1}(s) \\ \Phi \\ \Phi \\ W_{p_{21}}(s) \end{bmatrix} \\ \begin{bmatrix} -R_{p_1}(s) & \Phi & \Phi & W_{p_{12}}(s) \end{bmatrix} & \begin{bmatrix} W_{p_{11}}(s) \end{bmatrix} \end{array}\right]$$

```
function [T] = lftl(G,K)
%
%  This function operates on POLPAC system descriptions stored in modes 0,
%  4..6, and 7..9.  T is the lower linear fractional transformation (LFT)
%  of G and K.
%
%  Mixed mode operations are handled with a preference of trajectories
%  over coefficients over roots.
%
%  Input Parameters:
%  -----------------
%
%    G      := System in forward path
%    K      := System in feedback path
%
%  Output Parameter:
%  -----------------
%
%    T      := Transmission system ( r -> y )
%
%  Explanation:
%  ------------
%
%                   ---------------
%       r ------>|  G11   G12   |---------> y
%            --->|  G21   G22   |----
%            |      ---------------      |
%            |                           |
%            |          -------          |
%            --------|  K   |<--------
%                       -------
%
push('LFTL')
global debg
%
%  Check for necessary conversions first
%
[G,K] = mixmod(G,K);
%
%  Start by unpacking the structural information
%
[modG,lcolG,dtypG,stypG] = unpack(G(1,1));
[modK,lcolK,dtypK,stypK] = unpack(K(1,1));
logcol = max([lcolG,lcolK]);
[pG,mG] = logdim(G);
[pK,mK] = logdim(K);
domG = domain(G);
%
%  Check for consistency
%
if debg == 3,
  if modG > 0,
    if modG < 4,
      disp('LFTU: Error - Operates on system modes only'),
      abort,
    end,
  end,
  if stypK ~= stypG,
    disp('LFTL: Error - Incompatible system types'),
    abort,
```

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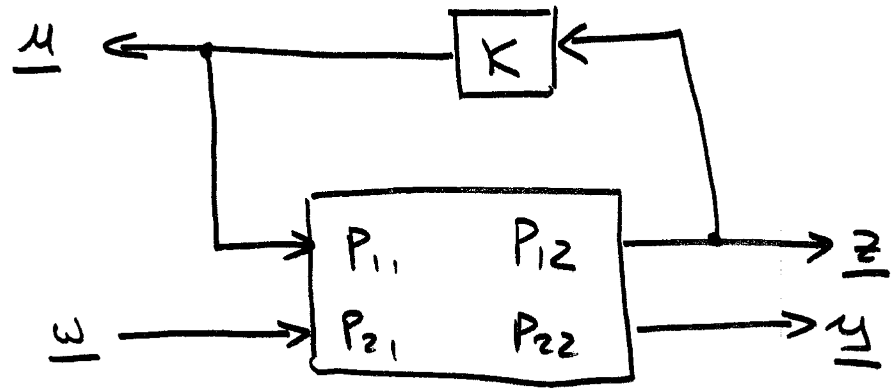
  end,
end
if debg >= 2,
  if mG <= pK,
    disp('LFTL: Error - Incompatible number of rows/columns'),
    abort,
  end,
  if pG <= mK,
    disp('LFTL: Error - Incompatible number of rows/columns'),
    abort,
  end,
end
%
%  Branch depending on the mode of the system.
%
if modG == 0,
  %
  %  Is a state-space description
  %
  [Ag,Bg,Cg,Dg] = oldsyst(G);
  [Ak,Bk,Ck,Dk] = oldsyst(K);
  %
  %  Extract the four subsystems.
  %
  Bg1 = Bg(:,1:mG-pK);
  Bg2 = Bg(:,mG-pK+1:mG);
  Cg1 = Cg(1:pG-mK,:);
  Cg2 = Cg(pG-mK+1:pG,:);
  Dg11 = Dg(1:pG-mK,1:mG-pK);
  Dg12 = Dg(1:pG-mK,mG-pK+1:mG);
  Dg21 = Dg(pG-mK+1:pG,1:mG-pK);
  Dg22 = Dg(pG-mK+1:pG,mG-pK+1:mG);
  DkDg22 = Dk*Dg22;
  DD = eye(size(DkDg22)) - DkDg22;
  %
  %  Check whether the problem is well-posed
  %
  if norm(DD) < 1000*eps,
    disp('LFTL: Error - System is ill-posed'),
    abort,
    return,
  end,
  %
  %  Now compute the feedback structure
  %
  DDinv = inv(DD);
  Bg2DDi = Bg2*DDinv;
  BkDg22DDi = Bk*Dg22*DDinv;
  DkCg2 = Dk*Cg2;
  DkDg21 = Dk*Dg21;
  Bcl1 = Bg1 + Bg2DDi*DkDg21;
  Bcl2 = Bk*Dg21 + BkDg22DDi*DkDg21;
  Bcl = [ Bcl1 ; Bcl2 ];
  Acl11 = Ag + Bg2DDi*DkCg2;
  Acl12 = Bg2DDi*Ck;
  Acl21 = Bk*Cg2 + BkDg22DDi*DkCg2;
  Acl22 = Ak + BkDg22DDi*Ck;
  Acl = [ Acl11 , Acl12 ; Acl21 , Acl22 ];
  Dg12DDi = Dg12*DDinv;
  Dcl = Dg11 + Dg12DDi*DkDg21;
```

```
  Ccl1 = Cg1 + Dg12DDi*DkCg2;
  Ccl2 = Dg12DDi*Ck;
  Ccl = [ Ccl1 , Ccl2 ];
  T = newsyst(Acl,Bcl,Ccl,Dcl,stypG);
  %
  %  Get rid of uncontrollable/unobservable modes
  %
  T = minreals(T);
  pull,
  return,
end
%
if modG == 4 | modG == 5 | modG == 6,
  %
  %  Is a transfer function matrix
  %  Extract the four submatrices.
  %
  G11 = gget(G,1:pG-mK,1:mG-pK);
  G12 = gget(G,1:pG-mK,mG-pK+1:mG);
  G21 = gget(G,pG-mK+1:pG,1:mG-pK);
  G22 = gget(G,pG-mK+1:pG,mG-pK+1:mG);
  %
  %  Now compute: T = G11 + G12*K*(I - G22*K)\G21
  %
  Aux = multg(G22,K);
  I = eyep(mK,mK,modG,logcol,stypG,domG);
  Aux = sub(I,Aux);
  %
  %  Check whether system is well-posed
  %
  [G2,P2] = strictprop(Aux);
  ord = order(P2);
  if ord == 0,
    gn = gain(P2);
    if gn == 0,
      disp('LFTL: Error - System is ill-posed'),
      abort,
      return,
    end,
  end,
  Aux = invg(Aux);
  Aux = multg(Aux,G21);
  Aux = multg(K,Aux);
  Aux = multg(G12,Aux);
  T = addg(G11,Aux);
  %
  %  Get rid of uncontrollable/unobservable modes
  %
  T = reduceg(T);
  pull,
  return,
end
%
if modG == 7 | modG == 8 | modG == 9,
  %
  %  Is a polynomial matrix system description
  %
  [Pg,Qg,Rg,Wg] = pms2plm(G);
  [Pk,Qk,Rk,Wk] = pms2plm(K);
  %
```

```
  %  Extract the four subsystems
  %
  [nQ,mQ] = logdim(Qg);
  [nR,mR] = logdim(Rg);
  Qg1 = gget(Qg,1:nQ,1:mG-pK);
  Qg2 = gget(Qg,1:nQ,mG-pK+1:mG);
  Rg1 = gget(Rg,1:pG-mK,1:mR);
  Rg2 = gget(Rg,pG-mK+1:pG,1:mR);
  Wg11 = gget(Wg,1:pG-mK,1:mG-pK);
  Wg12 = gget(Wg,1:pG-mK,mG-pK+1:mG);
  Wg21 = gget(Wg,pG-mK+1:pG,1:mG-pK);
  Wg22 = gget(Wg,pG-mK+1:pG,mG-pK+1:mG);
  %
  %  Plug closed loop polynomial matrices together
  %
  [qG,qG] = logdim(Pg);
  [qK,qK] = logdim(Pk);
  qT = qG + qK + mK + pK;
  pT = pG - mK;
  mT = mG - pK;
  Pcl = zerop(qT,qT,modG-6,logcol,stypG,domG);
  Im = eyep(mK,mK,modG-6,logcol,stypG,domG);
  Ip = eyep(pK,pK,modG-6,logcol,stypG,domG);
  Pcl = put(Pcl,Pg,1:qG,1:qG);
  Pcl = put(Pcl,Qg2,1:qG,qT-pK+1:qT);
  Pcl = put(Pcl,Pk,qG+1:qG+qK,qG+1:qG+qK);
  Pcl = put(Pcl,Qk,qG+1:qG+qK,qG+qK+1:qT-pK);
  Pcl = put(Pcl,minus(Rk),qG+qK+1:qT-mK,qG+1:qG+qK);
  Pcl = put(Pcl,Wk,qG+qK+1:qT-mK,qG+qK+1:qT-pK);
  Pcl = put(Pcl,minus(Ip),qG+qK+1:qT-mK,qT-pK+1:qT);
  Pcl = put(Pcl,minus(Rg2),qT-mK+1:qT,1:qG);
  Pcl = put(Pcl,minus(Im),qT-mK+1:qT,qG+qK+1:qT-pK);
  Pcl = put(Pcl,Wg22,qT-mK+1:qT,qT-pK+1:qT);
  Qcl = zerop(qT,mT,modG-6,logcol,stypG,domG);
  Qcl = put(Qcl,Qg1,1:qG,1:mT);
  Qcl = put(Qcl,Wg21,qT-mK+1:qT,1:mT);
  Rcl = zerop(pT,qT,modG-6,logcol,stypG,domG);
  Rcl = put(Rcl,Rg1,1:pT,1:qG);
  Rcl = put(Rcl,minus(Wg12),1:pT,qT-pK+1:qT);
  Wcl = Wg11;
  %
  %  Make a new polynomial matrix system
  %
  T = plm2pms(Pcl,Qcl,Rcl,Wcl);
  %
  %  Get rid of uncontrollable/unobservable modes
  %
  T = minrealps(T);
  pull,
  return,
end
%
return
```

Analogous:



$$\left| \begin{aligned} \underline{z} &= P_{11} \cdot \underline{u} + P_{12} \cdot \underline{w} \\ \underline{y} &= P_{21} \cdot \underline{u} + P_{22} \cdot \underline{w} \\ \underline{u} &= K \cdot \underline{z} \end{aligned} \right|$$

$$\Rightarrow \underline{u} = K \cdot \underline{z} = K \cdot P_{11} \cdot \underline{u} + K \cdot P_{12} \cdot \underline{w}$$

$$\Rightarrow \left[I - K \cdot P_{11} \right] \underline{u} = K \cdot P_{12} \underline{w}$$

$$\Rightarrow \underline{u} = \left[I - K \cdot P_{11} \right]^{-1} \cdot K \cdot P_{12} \underline{w}$$

$$\Rightarrow \underline{y} = \left[P_{22} + P_{21} \cdot \left[I - K \cdot P_{11} \right]^{-1} \cdot K \cdot P_{12} \right] \underline{w}$$

$$\Rightarrow \underline{\underline{\mathcal{F}_u(P,K) = P_{22} + P_{21} \left[I - K \cdot P_{11} \right]^{-1} K \cdot P_{12}}}$$

or:

$$\underline{z} = P_{11} \cdot \underline{u} + P_{12} \cdot \underline{w}$$

$$= P_{11} \cdot K \cdot \underline{z} + P_{12} \cdot \underline{w}$$

$$\Rightarrow [I - P_{11} \cdot K]\, \underline{z} = P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{z} = [I - P_{11} \cdot K]^{-1} \cdot P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{u} = K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12} \cdot \underline{w}$$

$$\Rightarrow \underline{y} = \left[P_{22} + P_{21} \cdot K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12} \right] \underline{w}$$

$$\Rightarrow \mathcal{F}_u(P, K) = P_{22} + P_{21} \cdot K \cdot [I - P_{11} \cdot K]^{-1} \cdot P_{12}$$

$\mathcal{F}_u(P, K)$ is the <u>upper linear fractional transform</u> of P fed back by K.

In the time domain:

$$P(s) = \left[\begin{array}{c|cc} A_p & B_{p_1} & B_{p_2} \\ \hline C_{p_1} & D_{p_{11}} & D_{p_{12}} \\ C_{p_2} & D_{p_{21}} & D_{p_{22}} \end{array}\right]$$

$$K(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

$$\left|\begin{array}{l} \dot{\underline{x}}_p = A_p \underline{x}_p + B_{p_1} \cdot \underline{u} + B_{p_2} \cdot \underline{w} \\ \underline{z} = C_{p_1} \cdot \underline{x}_p + D_{p_{11}} \underline{u} + D_{p_{12}} \underline{w} \\ \underline{y} = C_{p_2} \cdot \underline{x}_p + D_{p_{21}} \underline{u} + D_{p_{22}} \underline{w} \\ \dot{\underline{x}}_K = A_K \underline{x}_K + B_K \underline{z} \\ \underline{u} = C_K \underline{x}_K + D_K \underline{z} \end{array}\right|$$

$$\underline{u} = C_K \underline{x}_K + D_K C_{p_1} \underline{x}_p + D_K D_{p_{11}} \underline{u} + D_K D_{p_{12}} \underline{w}$$

$$\Rightarrow \underbrace{(I - D_K D_{p_{11}})}_{DD} \underline{u} = D_K C_{p_1} \underline{x}_p + C_K \underline{x}_K + D_K D_{p_{12}} \underline{w}$$

$$\Rightarrow \underline{u} = DD^{-1} \cdot D_K C_{p_1} \underline{x}_p + DD^{-1} \cdot C_K \underline{x}_K + DD^{-1} \cdot D_K D_{p_{12}} \underline{w}$$

$$\dot{\underline{x}}_p = \left[A_p + B_{p_1} \cdot DD^{-1} \cdot D_k C_{p_1}\right] \underline{x}_p + \left[B_{p_1} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k + \left[B_{p_2} + B_{p_1} \cdot DD^{-1} \cdot D_k D_{p_{12}}\right] \underline{w}$$

$$\dot{\underline{x}}_k = \left[B_k C_{p_1}\right] \underline{x}_p + A_k \underline{x}_k + \left[B_k D_{p_{12}}\right] \underline{w} + \left[B_k D_{p_{11}}\right] \underline{u}$$

$$\Rightarrow \dot{\underline{x}}_k = \left[B_k C_{p_1} + B_k D_{p_{11}} \cdot DD^{-1} \cdot D_k C_{p_1}\right] \underline{x}_p + \left[A_k + B_k D_{p_{11}} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k + \left[B_k D_{p_{12}} + B_k D_{p_{11}} \cdot DD^{-1} \cdot D_k D_{p_{12}}\right] \underline{w}$$

$$\underline{y} = \left[C_{p_2} + D_{p_{21}} \cdot DD^{-1} \cdot D_k C_{p_1}\right] \underline{x}_p + \left[D_{p_{21}} \cdot DD^{-1} \cdot C_k\right] \underline{x}_k + \left[D_{p_{22}} + D_{p_{21}} \cdot DD^{-1} \cdot D_k D_{p_{12}}\right] \underline{w}$$

$$\Rightarrow \mathcal{F}_u(P,K) = \left[\begin{array}{cc|c} (A_p + B_{p_1} DD^{-1} D_k C_{p_1}) & (B_{p_1} DD^{-1} C_k) & (B_{p_2} + B_{p_1} DD^{-1} D_k D_{p_{12}}) \\ (B_k C_{p_1} + B_k D_{p_{11}} DD^{-1} D_k C_{p_1}) & (A_k + B_k D_{p_{11}} DD^{-1} C_k) & (B_k D_{p_{12}} + B_k D_{p_{11}} DD^{-1} D_k D_{p_{12}}) \\ \hline (C_{p_2} + D_{p_{21}} DD^{-1} D_k C_{p_1}) & (D_{p_{21}} DD^{-1} C_k) & (D_{p_{22}} + D_{p_{21}} DD^{-1} D_k D_{p_{12}}) \end{array}\right]$$

$$
\left\{
\begin{aligned}
&P_p(s)\cdot \underline{x}_p(t) = Q_{p_1}(s)\cdot \underline{u}(t) + Q_{p_2}(s)\cdot \underline{w}(t)\\
&\quad \underline{z}(t) = R_{p_1}(s)\cdot \underline{x}_p(t) + W_{p_{11}}(s)\cdot \underline{u}(t) + W_{p_{12}}(s)\cdot \underline{w}(t)\\
&\quad \underline{y}(t) = R_{p_2}(s)\cdot \underline{x}_p(t) + W_{p_{21}}(s)\cdot \underline{u}(t) + W_{p_{22}}(s)\cdot \underline{w}(t)\\
&P_k(s)\cdot \underline{x}_k(s) = Q_k(s)\cdot \underline{z}(t)\\
&\quad \underline{u}(t) = R_k(s)\cdot \underline{x}_k(t) + W_k(s)\cdot \underline{z}(t)
\end{aligned}
\right.
$$

$$
\Rightarrow
\left\{
\begin{aligned}
&P_p(s)\cdot \underline{x}_p(t) - Q_{p_1}\cdot \underline{u}(t) = Q_{p_2}(s)\cdot \underline{w}(t)\\
&P_k(s)\cdot \underline{x}_k(t) - Q_k(s)\cdot \underline{z}(t) = \emptyset\\
&-R_k(s)\cdot \underline{x}_k(t) - W_k(s)\cdot \underline{z}(t) + \underline{u}(t) = \emptyset\\
&-R_{p_1}(s)\cdot \underline{x}_p(t) + \underline{z}(t) - W_{p_{11}}(s)\cdot \underline{u}(t) = W_{p_{12}}(s)\cdot \underline{w}(t)\\
&\underline{y}(t) = R_{p_2}(s)\cdot \underline{x}_p(t) + W_{p_{21}}(s)\cdot \underline{u}(t) + W_{p_{22}}(s)\cdot \underline{w}(t)
\end{aligned}
\right.
$$

Let: $\underline{\xi} = \begin{bmatrix} \underline{x}_p(t) \\ \underline{x}_k(t) \\ -\underline{z}(t) \\ -\underline{u}(t) \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} P_p(s) & \phi & \phi & Q_{p_1}(s) \\ \phi & P_k(s) & Q_k(s) & \phi \\ \phi & -R_k(s) & W_k(s) & -I \\ -R_{p_1}(s) & \phi & -I & W_{p_{11}}(s) \end{bmatrix} \underline{\xi}(t) = \begin{bmatrix} Q_{p_2}(s) \\ \phi \\ \phi \\ W_{p_{12}}(s) \end{bmatrix} \underline{w}(t)$$

$$\underline{y}(t) = \begin{bmatrix} R_{p_2}(s) & \phi & \phi & -W_{p_{21}}(s) \end{bmatrix} \underline{\xi}(t) + \begin{bmatrix} W_{p_{22}}(s) \end{bmatrix} \underline{w}(t)$$

is a polynomial matrix system representation of $\mathcal{F}_u(P,K)$.